



# NEWSLETTER

*Operational Research Society of New Zealand (Inc.)*



JUNE 1970

VOL. 5. NO. 3

## ANNUAL CONFERENCE

THE OPTIMUM PROVISION OF BURIED TELEPHONE CABLE      M.J. TARRANT  
Talk given at April ORSOC meeting

A NEW COMMITTEE STRUCTURE FOR THE SOCIETY      G.A. VIGNAUX

## COUNCIL NOTES

## INVITATION

INTERNATIONAL FEDERATION OF OPERATIONAL RESEARCH  
SOCIETIES      L.F. JACKSON

ANNUAL CONFERENCE

ABSTRACTS

These abstracts replace the two similar ones in the N.Z. Statistician, 5,1.

EXPERIENCES WITH A PROPRIETARY SIMULATION PROGRAM.

H.B. Moore, Operations Research Analyst, N.Z. Forest Products Limited, Tokoroa.

N.Z. Forest Products Limited have purchased the 'FOCUS' simulation program from the P-E Consulting Group of London. The program is written in Fortran for an IBM 1130 computer, but versions are available for most computers with Fortran compilers.

The main advantage of FOCUS over other simulation packages is its simplicity. A few days of training (and no knowledge of Fortran) can give reasonable proficiency in the system.

A description of the system, together with examples and case studies, is given.

THE PAPER TRIM PROBLEM.

C.D. Johanson, Operations Research Analyst, N.Z. Forest Products Limited.

The problem of fitting orders for paper reels across the parent reel from a paper machine to minimise waste is a well known application of linear programming.

N.Z. Forest Products Limited now minimise trim loss on certain grades of reeled paper using two package programs on the IBM 1130 computer. One uses linear programming and the Gilmore-Gomory knapsack algorithm. The other uses heuristic methods similar to those of a human scheduler. The advantages and disadvantages of each approach are discussed.

No program is available for the more difficult case of minimising trim loss on sheets. Work is in progress on this problem, using a combination of heuristic methods and linear programming.

OPERATIONAL RESEARCH SOCIETY SONG.

$\frac{(OR)^2}{2}$  \*

Hark! the O.R. workers sing  
"Quick and dirty is the thing".  
The day of "slow but sure" is past -  
We need returns and need them fast.  
Pave the way with simple tools;  
Leave the slow techniques to fools  
Who fail to see their proper place  
Is rather later in the race.

Then when you've opened up the field,  
Complicate for greater yield.  
It may be fun, it may seem best  
To publish sooner than the rest;  
It may be "with-it" - but come off it!  
What we want is greater profit!

by Max I. Mizer

\*  $\frac{(OR)^2}{2}$  =  $\int_0^{OR} ORd(OR)$  or the result of integrating OR with respect to itself up to its own limit.

# THE OPTIMUM PROVISION OF BURIED TELEPHONE CABLE

## An Application of Inventory Control

M.J. TARRANT  
(N.Z. Post Office)

This paper is a brief summary of investigations carried out for the special topic in Operations Research for the Information Science Course, Victoria University, 1969. The paper deals briefly with the ways in which various classical inventory control models can be adapted to a plant provision problem.

### Background

The N.Z. Post Office installs and maintains a vast network of buried cable in order to provide telephone service to subscribers. For each telephone subscriber a unique pair of wires is required in these cables from the subscriber's premises to the local telephone exchange. The investment in cable plant is currently expanding at the rate of some \$4M per year.

### Inventory System

It is clear that as demand for telephone service in a particular area grows, more and more cable pairs are required. As it is impractical to provide these pairs one at a time as required, an 'inventory' of spare pairs must be maintained to meet the demand as it arises. In common with all inventory problems, the correct 'order quantity' must be determined to effect an economic balance between the set up costs, in this case cost of excavating the road and installation cost etc., and the inventory holding cost, i.e. the interest incurred on non revenue earning capital.

As a first step, the classical lot size formula could be applied

$$q = \left( \frac{2rK}{hv} \right)^{\frac{1}{2}}$$

where

r is annual subscriber demand  
K is total fixed cost/foot  
h is interest on unused plant  
v is value of one cable pair/foot

This model is far from representative of the real situation for a number of reasons. These are:

- (1) The demand rate is not usually constant. (This implies linear growth).
- (2) The lot size is restricted by the range of available cable sizes.
- (3) The cost/pair is not directly proportional to the number of pairs in the lot.
- (4) The time scale for cable provision is of the order of 20 or more years, hence the time value of money or discount rate is a relevant factor not covered by the simple model.

It is possible to modify the lot size formula to accommodate some of these deficiencies in the following manner:

..../

- (1) The fact that the demand is not linear can be roughly allowed for by estimating the probable demand during the life of the "first in cable" - usually 8 - 10 years, and assuming linear growth over that period. This is by no means a precise method but tests of the model showed that it was relatively insensitive to forecast errors beyond the life of the "first in cable".
- (2) If we place a constraint on the optimum lot size to be, say, a multiple of 50 pairs (this will cover nearly all available sizes), then clearly the cost of the optimum policy  $C(q_0) \leq C(q_0 + 50)$

and  $C(q_0) \leq C(q_0 - 50)$   
 substituting the actual cost

$$\frac{h v q_0}{2} + K r / q_0$$

we obtain an optimality condition

$$q_0 (q_0 - 50) \leq 2Kr / hv \leq q_0 (q_0 + 50)$$

which reduces to

$$\gamma = \text{int} \left\{ \frac{1 + \sqrt{1 + 4J}}{2} \right\}$$

where  $J = 2Kr/hv \cdot 50^2$   
 and  $\gamma = q_0/50$  - an integer

- (3) The true cost of cable was investigated and it itself found to exhibit both fixed and variable components and could be written with negligible error as  $c = aq + b$ . Hence the fixed component can be lumped with the trenching cost  $K$  and the above model remains unaltered.

The above model, while very simple to apply, cannot however take account of the time value of money and with consideration of the time scale involved it must reluctantly be rejected as unsuitable.

#### Dynamic Models.

Because of the long time scale (20 + years) during which time there may be only 3 or 4 'replenishments' of cable pairs, it is really only valid to compare policies which operate over the long time scale and have provision for discounting future expenditure. We must now compare strategies of sets of decisions rather than single decision models such as the lot size model.

The most suitable method for optimising these multiple decision processes is dynamic programming. Three quite different dynamic programming approaches have to be considered, all with merits and demerits. These are:

- (A) An adaption of Wagner and Whitin's Inventory Algorithm (ref. 1)
- (b) Discrete state numerical algorithm.
- (C) Variable stage algorithm (Knapsack).

..../

A. The Wagner and Whitin Algorithm.

This method is very efficient, requiring the evaluation of a maximum of 210 possible solutions in the optimisation process. A computer program of this algorithm took approximately 6 seconds on an Elliott 503 computer. The formulation of the appropriate model does however, require some simplifying assumptions and these are:

- (1) shortages are not permitted.
- (2) Zero leadtime.
- (3) New cables are only installed if there are no spare pairs remaining.
- (4) Each cable is sufficient for a whole number of review periods. (This can be seen to be optimal because if a policy states that a cable of K pairs is installed when  $\gamma$  are still in hand, it will always be cheaper to have installed  $\gamma$  less the previous time.
- (5) The constraint on available size of cables is lapsed.

The model has the following notation:

N = number of periods in the planning horizon

$r_i$  = demand during period i

S = trenching cost/foot

$\alpha$  = discount factor

h = holding cost/unit carried forward to the next period.

Note Because of the discount factor it is no longer valid to include the interest charge in the holding cost. h now reflects only those 'real' charges such as maintenance.

Because of the requirement of zero spares when a new cable is installed and the whole period supply criterion, any cable containing  $q_K$  pairs installed to last just until period i must exactly contain

$$q_K = \sum_{j=K}^i r_j \text{ pairs}$$

Also if  $\mu$  is some intermediate period between K and i then at the close of period  $\mu$  we will be left with exactly

$$I_\mu = q_K - \sum_{v=K}^{\mu} r_v \text{ pairs}$$

This is because, by definition, no pairs were carried over into period K, an instalment period. Notice the  $I_\mu$  is solely determined by the demand characteristic r. The holding cost for the period between K and i, is just

$$h \sum_{\mu=K}^i I_\mu$$

Once again, because there can be no spares carried over into period K, the minimum cost up to the end of period i must be the minimum cost up to the beginning of period K,  $M_{K-1}$ , plus the cost of one further instalment and holding charges.

That is 
$$M_i = \min_K [ M_{K-1} + S + h \sum_{\mu=K}^i I_\mu ]$$

...../

It is a simple matter to introduce the discount rate  $\alpha$  by replacing the trenching cost  $S$  by the total instalment cost  $R_K$  giving finally

$$M_i = \frac{\min}{K} [M_{K-1} + \alpha^{K-1} (R_K + h \sum_{\mu=K}^i I_\mu)]$$

where both  $R_K$  and  $I_\mu$  can be determined by knowledge of demand  $r$ .

The computation procedure is as follows:-

- (a) set  $i = 1$ ,  $M_0 = 0$ .
- (b) Calculate the cost function for  $K = 1, i - 1$ , etc., until the holding cost is greater than the set up cost or  $K = i$ .
- (c) Retain the minimum  $M_K$  and increase  $i$  by 1.

What this enumeration process does is as follows:-

- (a) starting at year 1 we have only one choice - that is, provide pairs for year 1.
- (b) In year 2, we can do either:
  - (i) Use year one's pairs and provide only for year 2.
  - (ii) Provide initially (year 1) enough for both years 1 and 2.
- (c) In year 3, we can do either:
  - (i) Provide year 3 only and use the cheapest policy from year 2.
  - (ii) Provide year 3 and 2 and use previous year 1 policy.
  - (iii) Provide years 1, 2, 3 all at the outset.

From this it can be seen to be a true dynamic programming 'learning' process although the enumeration is somewhat unusual.

This process does not however, enable the automatic elimination of policies which require non-standard cable sizes, as do the following two methods.

### B. Discrete State Dynamic Model.

Considering the cable under review at any time period as containing 0, 1, 2, ... 100 spare pairs (or multiples) and considering each year of the planning horizon as defining one possible stage in a multi-decision process, then at each year (stage) a decision must be made to install one of 15 possible cables (including zero), based on the state (spare pairs) of the system at that stage. However, the problem cannot be solved simply by advancing from year to year as the current state in any year is governed by those cables which were installed earlier, but by ordering the decision process in reverse and taking advantage of the fact that discounted value 20 years in the future is not worth much today, a simple numerical procedure enables speedy calculation of the optimal strategy.

- (1) Set up a cost matrix of 100 columns representing the value of having 1, 2, 3, ... 100 spare pairs in each year, with all elements in year 20 equal to zero.
- (2) If previous optimal policies cause the state of the system to be  $j$  spare pairs at year 19, then we must find the cheapest cable which together with the  $j$  spare pairs will just meet the demand until year 20. This is the value of state  $j$ ; stage 19. This is found for all  $j = 1$  to 100.

..../

- (3) For year 18, if the previous optimum causes entry at state K, then we must find the cheapest combination of new cable plus the value of entering stage 19 with the appropriate number of spare pairs. In this case  $K + \text{cable} - \text{demand year 18}$ . This is repeated for all K 1 to 100.
- (4) Each stage is considered similarly, each time taking the cost of each of 15 cables plus the entry value (discounted) to the next stage. The minimum in each case becomes the value of that state in the current stage.

Mathematically this can be written as a recursion equation:

$$V(n,i) = \min_x [ C^x + V(n-1, i+K^x-D(N-n+1)) ]$$

where  $V(n,i)$  is the value of occupying state  $i$  (spare pairs) with  $n$  periods remaining in the horizon.

$C^x$  = cost associated with policy  $x$

$K^x$  = pairs provided by policy  $x$

$D_j$  = demand in period  $j$ .

A program on the Elliott 503 computer for this procedure took approximately 60 seconds demonstrating the larger number of possible alternatives investigated with this method.

In the worst case this method will evaluate 30,000 possible alternatives, still much less than the  $15^{20}$  possible combinations.

The method does however, have all the characteristics required such as variable costs, discounting, discrete cable sizes.

### C. Variable Stage or Knapsack Algorithm.

This is possibly the best method but was not developed fully or tried on test runs. However a brief description is as follows.

The problem is considered as a Knapsack of capacity equal to the 20 year subscriber demand. This knapsack must be filled with a number of discrete cables chosen from the 15 available cable sizes.

The problem is solved in the following way:

- (1) Consider that the knapsack is filled up to the "last in cable". It must now be decided which of the 15 cables should be the last in. This is equivalent to choosing in which of the 20 years the last instalment should be placed.

As there is no prior information, the cost of installing a last in cable for each of the 20 years of the horizon must be found. Each cable will be unique as the total remaining demand must be fully satisfied, so only 20 different costs need be examined. This is the initialisation step.

- (2) Now investigate the second last instalment. If this goes in year 19 then it can last only until year 20, i.e. one possible cable.

..../

If it is installed in year 18 there is the possibility of supplying only year 18 or both year 18 and 19, i.e., 2 possible cable sizes to which must be added the cost of a last instalment in year 19 or 20 and a minimum selected. So on for year 17, 3 alternatives; 16, 4 alternatives..... .., 0, 20 alternatives, a total of 210 alternatives.

At the completion of this set of calculation we have a cost matrix giving the minimum cost of the second last instalment placed in any year from 0 to 19. If the overall minimum is year 0 then this is the overall optimum policy. If not we repeat the procedure for a third to last instalment.

In general there will be fewer than four instalments in the optimum policy involving approximately 800 calculations as compared with 30,000 by the previous method.

It is expected that this method would be very fast, comparable with the Wagner and Whitin Algorithm. It enables the relevant features such as discrete cable sizes to be accounted for.

#### References

- (1) H.M. Wagner, T.M. Whitin: Dynamic Version of Economic Lot Size Model, Management Science Vol. 5, No. 1, Oct. 1958.

## A NEW COMMITTEE STRUCTURE FOR THE SOCIETY.

Council has recently set up a series of informal committees to advise Council on particular areas and in some cases to take over most of the working responsibility in those areas for both organisation and decision making. The idea is not to proliferate committees (which is the last thing an O.R. Society should do); it appeared to Council that there were a number of telling reasons why this decentralisation should be carried out at this time.

First the idea is to involve more ordinary members of the Society in the organisation and the decision making structure. This larger number of members should, we hope, be a source of fresh ideas and should perhaps enable anyone who is really interested, to make his ideas known and to have a chance of influencing the way the Society is run.

Another compelling reason why the decision was made now, is that the work required of members of Council seems to have increased sharply over the last year. For example, with our recent admission to IFORS, there will be an additional secretarial burden in communicating with IFORS and in receiving the IFORS abstracts which have to be distributed amongst full members. This was obviously too much to ask of an already well-worked secretary and this will be one of the tasks that the International Committee will be asked to carry out. Another advantage of the increased representation is that there will be an experienced, (but not exclusive) source of members for election to Council. More members will become involved in the operations of the Society leading to participatory democracy rather than merely representative democracy.

With the spreading of the load we should be able to give closer scrutiny to those matters which are really important to the Society and which up till now Council has had to look at at the same time as all the other matters which seem to come its way.

Council has, in this initial stage, decided that the membership of the committees shall in general be made up of a Chairman who is also a member of Council and invited members. The Council will retain its final responsibility for all decisions and actions.

If any member of the Society is particularly interested in one of the areas for which there is a committee and wishes to offer his services, I would ask him to contact the President. Volunteers are always to be welcomed.

The new committees, their members and a brief note of what they are for are listed below.

Membership Committee, D. Cook (Chairman), K. Hall, L.O'Brien  
(to handle all membership applications and to recruit new members).

Programme Committee, R. Wheeler (Chairman), H. Barr, C. Walker, E. Christiansen  
(to arrange all Society meetings, and in particular, the Annual Conference).

International Committee, L.F. Jackson (Chairman), P. König, J. O'Dea  
(to act as liaison with IFORS and other international bodies).

Publications Committee, B.K. Campbell (Chairman), M.J. Tarrant, L.E.O'Brien  
(to publish the Newsletter)

Public Relations Committee, P. Bielecki (Chairman), L.F. Jackson, B. Kaiser  
(to increase public knowledge of the Society).

G.A. Vignaux  
President

## C O U N C I L   N O T E S

Notes from Council meetings held November 28th 1969; February 3rd, 1970; March 3rd, 23rd; April 20th.

---

### COMMITTEES

It was agreed to set up committees to help Council in the following areas: Membership, Programme, International, Publications, Public Relations. See elsewhere in this newsletter for further details. Terms of reference for all the committees have been approved.

### VISIT TO NEW ZEALAND BY DR JOHN WALSH

Dr John Walsh, (an ex-president of ORSA) may visit New Zealand to lecture in 1971. Consideration was given to the possibility of helping sponsor this visit.

### INSTITUTE OF MANAGEMENT

The Institute asked that help be given to provide a series of evening lectures on "Introduction to OR".

### OR BULLETIN

It was agreed that the Society and the D.S.I.R. will prepare and produce a (single) bulletin promoting the application of OR to New Zealand problems. The costs of production will be borne by the D.S.I.R.

### ANNUAL BUDGET

This years budget has been prepared in a manner designed to show how much is being spent on the various services and activities of the Society. A net loss will be the predicted consequence of maintaining activities and services at last year's levels.

### LONG OVERDUE SUBSCRIPTIONS

It was decided to delete from the records members whose subs were more than 3 years overdue, unless payment was quickly made.

### RESIGNATION OF TREASURER

Mr Andrew Wallace, treasurer to the Society for the last 1.5 years resigned because he was taking a job in Christchurch. Council accepted his resignation with regret, and carried with acclamation a motion of appreciation of his services.

### NEW TREASURER

Mr Colin Walker was co-opted to Council and appointed as Treasurer.

...../

OR JOURNAL

Consideration was given to the idea of the Society producing a New Zealand OR Journal to act as a formal vehicle for New Zealand OR papers. It was thought to be a desirable long-term aim, but might be premature at present.

ANNUAL CONFERENCE

Speakers have been arranged for the annual conference. The President mentioned the possibility of charging a conference fee.

I N V I T A T I O N

This is an invitation to members to address meetings of the Operational Research Society.

This year the Programme Committee have had difficulty in arranging meetings. Apart from the occasion in July when we will join with the Statistical Association for the annual 3 day joint conference, only two firm dates have been fixed, for meetings of the Society, June 15th and July 20th.

If you have a paper you wish to present or know of someone who is willing to speak to the Society, please write and inform

The Chairman of the Programme Committee,  
R.C. Wheeler,  
89 Spencer Street,  
Crofton Downs,  
Wellington

or Dr. H. Barr,  
C/- Applied Mathematics Division,  
D.S.I.R.  
Wellington

or Mr E. Christiansen,  
C/- Philips Electrical Industries of N.Z. Ltd,  
181 - 195, Wakefield Street,  
Wellington.

## IFORS NEWS

The O.R. Society of N.Z. has just paid its first annual dues to IFORS. Members of our Society may wonder what is involved in membership of the Federation.

When the societies in the Federation approved our membership, they showed that they were willing to recognise the professional standing of the Full Members of our Society, for one of the central ideas of IFORS is that it is a federation of professional societies.

### IFORS and the United Nations Organisation

The current president of IFORS is Mr Alex Lee, formerly of Air Canada, and now with Rolls Royce. Among his main interests at present is to explore the needs for, and possible uses of, operational research in the activities of the United Nations Organisation and its agencies. In a recent statement he said,

"I believe that as an international organisation promoting the uses of rational scientific enquiry into the problems of large organisations and their management, IFORS has a responsibility to do whatever it can to assist the United Nations and its agencies in determining rational means for defining and solving their problems. As your elected President and representative, I made this point in my opening address to the national conference of one of the three founding societies of IFORS, namely the Operational Research Society, in England, last November. In the near future, I hope by courtesy of the Operational Research Society to be able to send you a copy of my speech.

I should like to be permitted as IFORS President, however, to say that, in my opinion, if IFORS fails to discharge its responsibilities towards the international community it will lose much of its significance and potential value."

### Conferences

When IFORS member societies designate a conference as an international conference of the federation, each member society is entitled to send 3 delegates, and further places may be available depending on vacancies. Two such conferences may be of special interest to members.

1. September, 1971. In Montreal, Operations Research in Banking and Finance.
2. April, 1971. International Cost Effectiveness Conference in Washington, D.C.

There are a number of other conferences we have been advised of in late 1970. Any others thought to be of particular interest will be advised in the Newsletter.

### International Abstracts

Full members of the Society will start receiving International Abstracts in Operational Research late in the year.

L.F. Jackson  
IFORS Representative.