Coordination in vehicle routing

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Abstract

A coordination point is a place that exists in space and time for the transfer of materiel and/or personnel and/or information between vehicles. The transfer may be initiated to accommodate payload, driver or vehicle requirements. This is in contrast to a node that exists for the transfer of goods between customer(s) and/or depot(s). Size, location or movement does not limit coordination points. They can be as large as a sub-depot or as small as a single letter drop-point; they can be strips of roads, areas of seas or deserts, or volumes of air space. They can be stationary, as in most land-based applications, or mobile as in the mid-air refuelling of aircraft. They have spatial and temporal characteristics that play a role in determining their classification, and can be serendipitous or pre-determined.

In this variation of the vrp, the payload of at least some vehicles is changed after they leave the depot, in a pure delivery scenario with identical vehicles that begin and end daily tasks at a common, single depot. The objectives are to reduce the total distance travelled and to reduce the size of the vehicle fleet, thereby reducing operating costs.

Loads can be re-deployed between the depot and the customer node by using coordination points that might involve split deliveries. Although such points can occur anywhere, this talk will concentrate on coordination at existing nodes.

Some effects of coordination are to reduce the need to return to the depot between routes for multi-routed vehicles; to allow vehicles to meet en route and to swap loads; or to act as interim steps in route improvement. This is demonstrated by reference to particular examples in both the Euclidean and rectilinear grid network environments and represents work in progress.

1 Introduction

In the standard vehicle routing problem, a depot houses a fleet of vehicles which deliver goods to and/or collect them from a set of customer nodes. The aim is to effectively and efficiently service all the nodes in the shortest time and/or distance, possibly with the smallest fleet size, with consideration given to the requirements of the customers, the number of depots and the characteristics of the fleet. There are many variations of this problem. In the coordination variation [1], the payload of at least some vehicles is changed after they leave the depot. This is done to reduce the total distance travelled or to reduce the size of the vehicle fleet. Temporal and spatial characteristics are interdependent. Coordination is investigated in the Euclidean plane and in a rectilinear grid network . In addition, a heuristic is presented which greatly reduces the size of a grid by eliminating non-essential edges.

2 Coordination Points

2.1 Classification

Coordination points are locations that are temporary sites used to hold goods dropped off by a vehicle and collected by another vehicle or by that same vehicle later. Coordination points can be at existing nodes, on part of a route or somewhere close to other routes. Load re-deployment at coordination points is commonly used by couriers swapping loads on the side of the road. Same-vehicle coordination points are used by stock vehicles that drop a trailer somewhere and collect it later. This not only reduces the overall distance travelled by the complete vehicle, it may improve access and allow the vehicle to use a track that the complete vehicle and trailer cannot. The advantages have to be weighed against the extra distance to retrieve the trailer.

The process of coordination might involve split deliveries. The use of split deliveries means that a customer receives its demand in parts, instead of in one piece, from more than one source - possibly the same vehicle at different times, or perhaps different vehicles at the same time or at different times. Not all types of load can be split: not all customers might want their load split. This gives rise to consideration of commodity classes and to customer types.

Practicable coordination point classifications are:

- serendipitous meeting on existing route
- pre-determined meeting on existing route
- post-determined meeting on existing route
- pre-determined detour
- post-determined detour
- sequential multiple vehicle at node
- simultaneous multiple vehicle at node
- sub-depot
- single vehicle off-route
- single vehicle drop-off (at node or existing route)
- mobile mop-up.

2.2 Reasons For The Existence Of Coordination Points

Coordination points are not just an interesting mathematical phenomenon; they exist in a variety of physical situations. They may exist under circumstances which include a combination of the following reasons:

- to reduce the total distance travelled by a vehicle fleet. Incorporating a coordination point enables the overall distance to be reduced, since common distances need not be traversed by more than a minimum number of vehicles.
- to reduce the size of a fleet. When using a coordination point increases the occupancy rate of a vehicle by allowing that vehicle to be used on more than one route in a work cycle, the size of the vehicle fleet is reduced. The occupancy rate is increased by having the vehicle more fully laden more of the time, or by re-using the vehicle on smaller routes.

- to increase the area of operations of the concern employing the fleet. A coordination point can allow vehicles to travel further because they do not have to travel over common ground. As a result, a company can increase its area of operations to take in a wider client base. In the case of mid-air re-fuelling of aircraft, the transferred item is aviation fuel, which allows the recipient plane to extend its point of no return.
- repair. The coordination point is the breakdown site.
- to enable high priority loads to return to the depot more rapidly than other loads. Higher priority loads can be transferred to vehicles which travel directly to the depot or to vehicles which travel more quickly because they visit only the high priority customers. Express couriers use this type of coordination point extensively.
- to enable high priority loads to be delivered more rapidly than other loads. This situation is similar to that above. The items transferred may be high priority or lower priority loads depending on the individual circumstances.
- to split the delivery vehicle. Heavy unladen trailers are expensive to carry. Similarly, trailers which are laden with goods which will not be needed in a specific region have no requirement to enter that region if they can be transferred to another vehicle or if they can be unhooked and later retrieved by the returning vehicle. Stock vehicle drivers use this method widely to avoid taking large trailers into some areas. Also, trailers can be unhooked to allow access for the main vehicle when it would not be able to use certain roads because of tight turns, small bridges and other hazards to navigation.
- to reduce operating costs. Operating costs include running costs of vehicles, purchase costs, rental costs and special taxes.
- to accommodate split deliveries/pickups. Splitting loads allows loads to be carried which can not fit into one vehicle.
- to accommodate demands which become available at different times. Individual customers sometimes have more than one request in each work cycle. These requests do not necessarily occur at the same time; some happen after the vehicle has been despatched to deal with the first one. Coordination points allow the customer to be serviced by fewer vehicles than the number of requests when some of those requests can be combined en route. Coordination points also allow later vehicles to deviate from their existing routes to service such customers without too much detriment to those previous routes.

3 Applications of Coordination Points

There are three main effects for a coordination point. It can be used

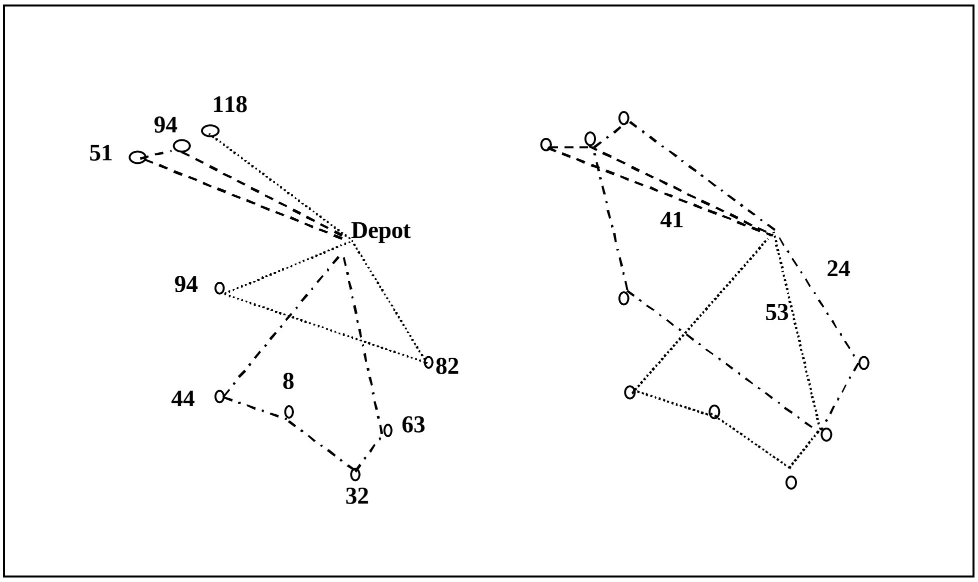
- to obviate the need for returning to the main depot for a multi-routed vehicle
- to swap loads between vehicles
- as an interim step to better a solution.

3.1 Obviating the Need for Returning to Depot

As the result of a routing schedule, one or more vehicles may service more than one route each. The length of one route is determined by the carrying capacity of the vehicle, the sum of the demands of the nodes allocated to it, and limitations on the length of a route or on the time of a route. A vehicle, is also limited by the maximum time it can be on the road or by the maximum distance it can travel without a break. It is sometimes possible for a particular vehicle to service more than one route without violating distance and timing restrictions. That is, a vehicle can travel one route, return to the depot, collect the load for another route and travel that route before its resources are exhausted. In a pure delivery scene, the distance involved in returning to the depot to collect that second (or subsequent) load is dead time, since no customer is being serviced. Coordination points are used to reduce or eliminate that dead time. This form of coordination applies to multi-routed vehicles. The load for the second route is dropped at a point that the first route can access; it collects the load and travels on the second route without having to return to the depot. The collection vehicle might need to return to depot inside the second route.

When coordination points are created in order to reduce the need for multi-routed vehicles to return to the depot, split deliveries can be incorporated. The load for at least part of the later route is accommodated on a second vehicle, which then drops these part-loads at a location which can be reached by the vehicle travelling the earlier route.

Consider the example below which shows a single depot, nine node system. Demand is shown beside each node in the left hand, four route system.

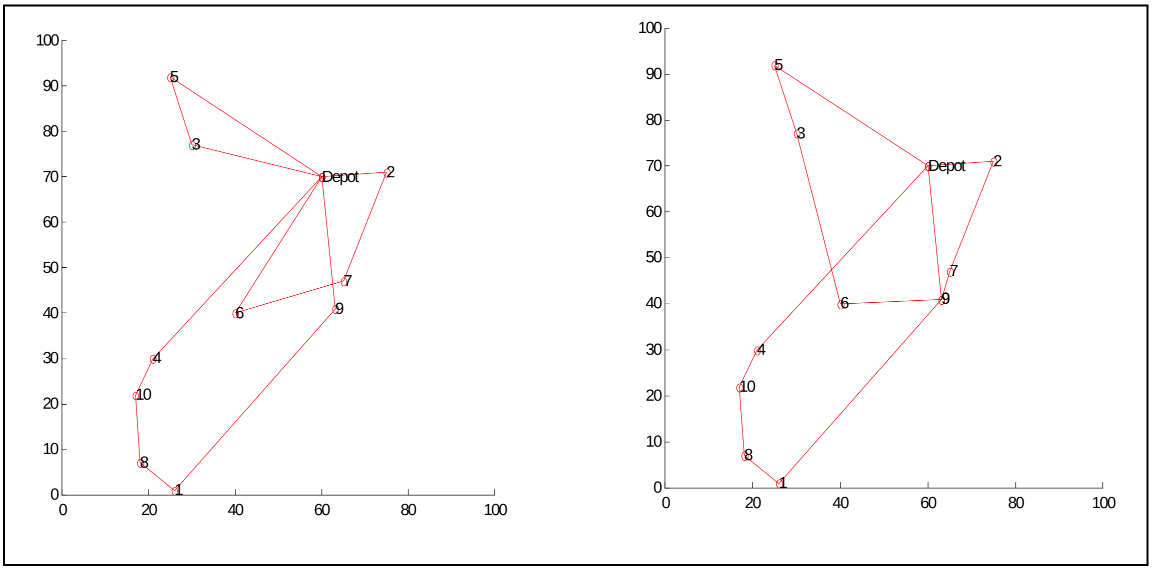


1. Before (left) and after (right) coordination to remove a return to depot.

Before coordination, three vehicles serviced the nodes using four routes. One of the vehicles returned to the depot after its first route to collect the load (118) for a second route. Using coordination, the demand of 118 for the node on a single-node route is split between the other three routes using spare capacity on the three vehicles. The split demands are shown in the right hand diagram. All final routes are clockwise. Two nodes become coordination points. At the first coordination point, two parts of the split load are combined on one vehicle which then travels to the second coordination point, the node immediately preceding the 118 node. At this coordination point the remainder of the load which has already been delivered is re-united with the other two split parts. Thus, the demand is carried split but delivered intact.

3.2 Swapping Loads

When sum(Total Load) ≤ Max Load × Fleet Size, the total load could possibly be shared between the vehicles without overloading any, if the actual demands permit it without breaking up any load. Consider the example below.



2. Before (left) and after (right) using coordination to swap loads.

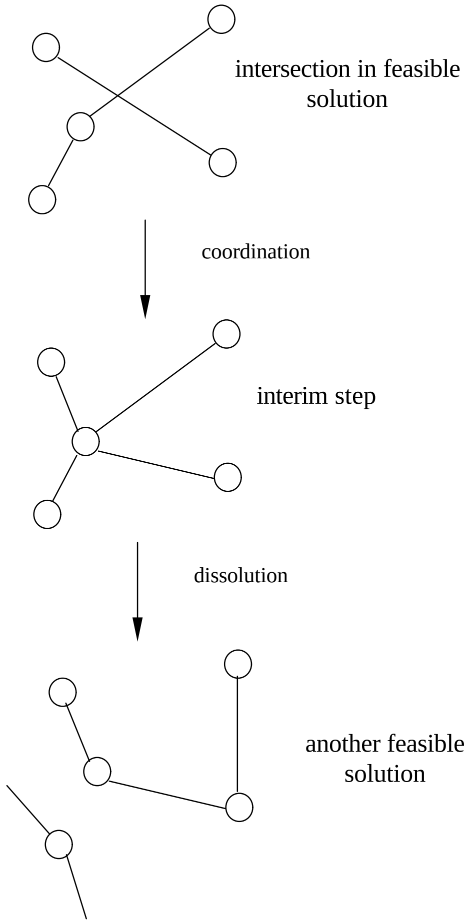
Node 9 becomes a coordination point. At the coordination point, vehicle 1 gives up demands for nodes 3 and 6, and collects demands for nodes 1, 10. A consequence of the load swapping is that the number of routes is reduced from three to two. The total distance is reduced by nearly 8%.

3.3 Coordination Point as Interim Step

Coordination points can be created which violate load and distance constraints. These coordination points are eliminated in later steps to produce routing structures with reduced fleet size or reduced total distance.

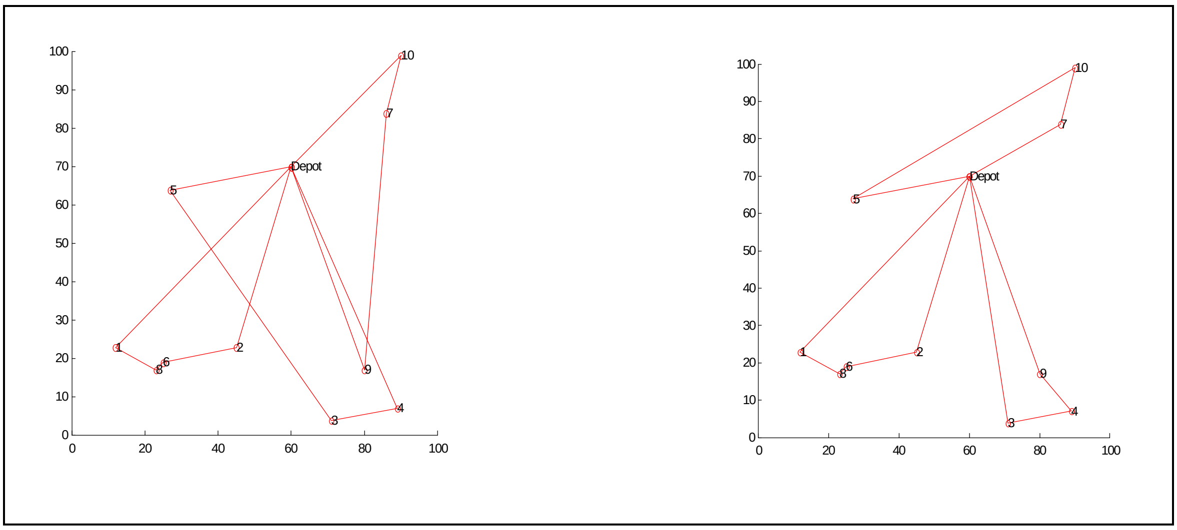
Assume that a coordination point is created by the intersection of two routes. At the coordination point, four part-routes impinge: the before- and after-encounter for each of the two routes. When the coordination point is an interim step, those four part-routes rearrange so that the coordination point is in only one of the resulting routes, not in both. This differs from the use of a coordination point to swap loads, where the coordination point appears in both routes.

Five nodes make up the interim coordination point system: the coordination point node, the node immediately preceding it and the node immediately following it on its scheduled route, the two nodes on the intersecting route that define the intersecting link.



3. Coordination as an interim step.

Consider the example below. The system on the left of the figure shows several nodes close to three intersections. After applying coordination, the system on the right has a reduced distance of 13%. Every route has been changed, even though the intersection involved only two of the routes.



4. Before (left) and after (right) using a coordination point as an interim step.

4 Coordination Hull

Regardless of the type of coordination, the coordination point is always located within a coordination hull. In the Euclidean scenario, the hull is a convex hull created by the nodes and depot. In the rectilinear grid network, the hull is created by the external perimeter of the reduced network, where all extraneous edges have been removed.

4.1 Network Reduction

Rectilinear grid networks are generated by forming a complete Manhattan graph and then deleting some edges. All edges are two-way. Each grid is defined by two 0/1 matrices: EH representing the horizontal edges and EV representing the vertical edges.

$$\text{EH(i, j)} = \begin{cases} 1 & \text{if there is an edge to the right of vertex (j, i)} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\text{EV(i, j)} = \begin{cases} 1 & \text{if there is an edge above vertex (i, j)} \\ 0 & \text{otherwise} \end{cases}$$

All changes to a particular grid are accompanied by minor changes to the contents of EH and EV without affecting the size of the matrices. The upper limit on the size of problem that can be tackled using this method of grid definition depends on the storage capacity of the computing device.

Nodes are located randomly at the vertices of the grid, rather than along edges. This simplifies the computations without loss of generality since a node that lies on an edge can be considered to lie a fraction of the length of the edge from either of the two vertices defining that edge. Edge location of nodes would introduce another set of variables into the computations made up of one of the end vertices defining the edge and the fractional displacement in a particular direction from the vertex. The underlying concepts would not change since vertex location can be considered as a special case of edge location in which the displacement along the edge is always zero.

A hull is formed by the nodes and depot(s) with regard to the edges connecting them. When a node is located on an outcrop of edges, those edges connecting that outcrop must be included within the hull. Only that part of the grid that falls inside the boundary of such a hull need be connected. As in the Euclidean case, coordination can occur only within the hull. All edges that are outside the hull are extraneous and can be eliminated from consideration by a process of grid reduction.

Grid reduction is the process of removing unnecessary edges from a rectilinear grid. An edge has value only if it touches an essential point or if it contributes to the shortest path between two essential points or if the vertices at either end provide a site for coordination.

The vertices of a grid can be classified according to the number of edges impinging upon them. A *cul-de-sac* is defined as a string of edges that does not lead to an intersection. A *No Choice String* is a string of edges that includes no intersections. Once a No Choice String is encountered, a vehicle is compelled to continue along it in the prescribed direction of travel until the No Choice String is finished. In some cases, a No Choice String may be replaced by an alternative route from one end of the No Choice String to the other. An *essential point* is a node or depot.

Number of Edges	Vertex Classification
0	Unused vertex
1	End of a cul-de-sac
2	Member of no-choice-string
3	3-way intersection
4	4-way intersection

Table 1. Classification of vertices.

The number of edges in contact with a vertex is found by summing the appropriate elements of the EH and EV matrices.

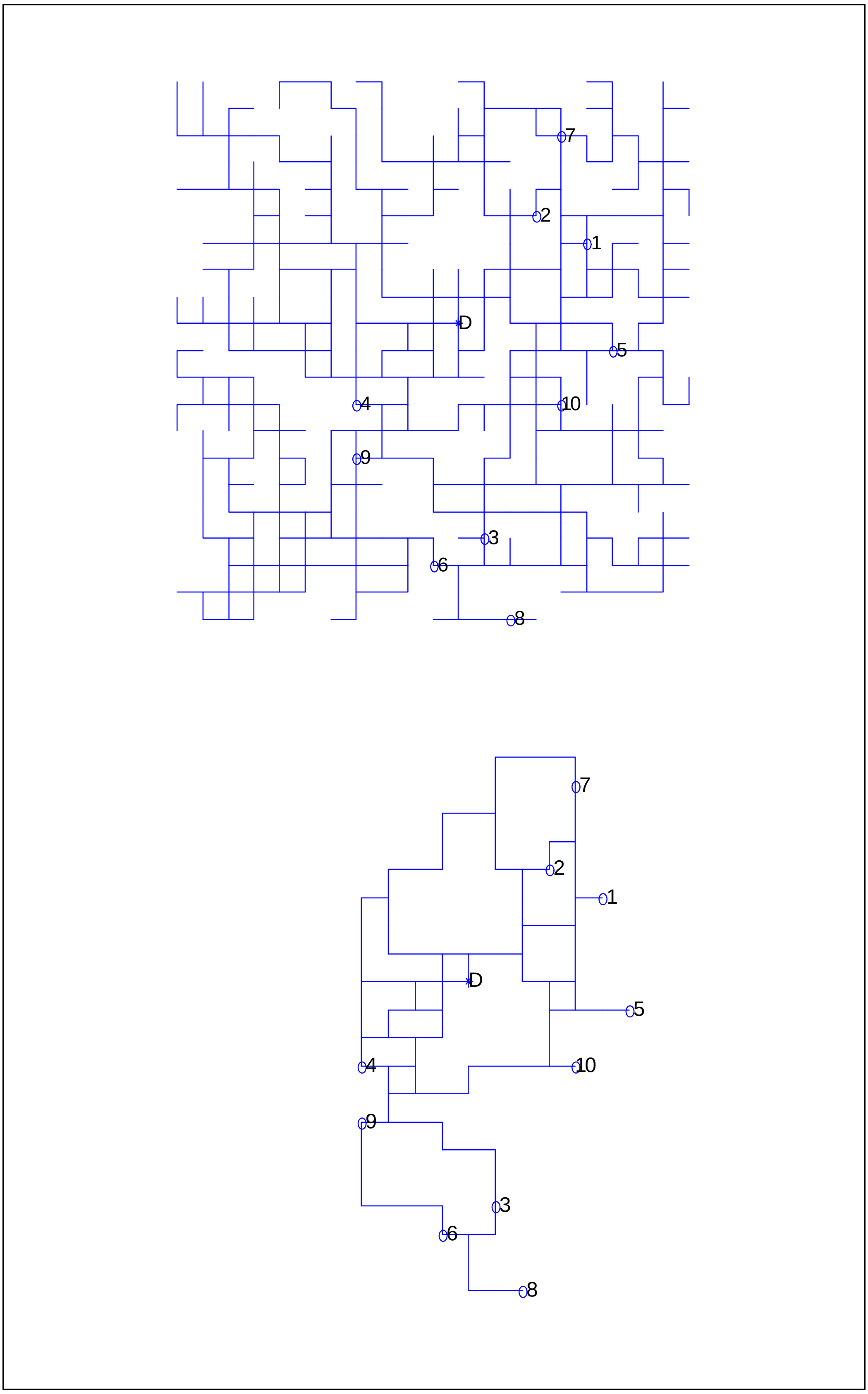
A cul-de-sac that does not include a node or depot will not be used as an alternative route or as a site for coordination since it does not lead to an essential point. It is superfluous to any consideration of vehicle routing and can be removed from the grid.

A No Choice String that does not include a node or depot and that is no longer than an alternative route between its ends can always be replaced by another route of the same length or less. It is also superfluous to the overall routing scheme.

As cul-de-sacs and no-choice-strings are removed from the grid, the status of vertices adjacent to removed edges changes, inducing minor changes in EH and EV. These vertices need to be re-checked until they do not exhibit changing status.

Initially, only those edges not involving essential points are examined for removal. When all such edges have been dealt with, the edges involving essential points are considered. The process is repeated until all extraneous edges have been removed.

Reduction of a single depot/ten-node system on a sparse grid typically decreases the number of edges from about 440 to around 100, depending on the location of the essential points and the structure of the initial grid. Not every remaining edge will be involved in a shortest path between essential points. Grid reduction is an effective means of producing the coordination hull since all edges outside the hull are removed. Once the grid has been reduced, routes can be constructed and checked for improvement by coordination more easily, since there are fewer edges to consider.



5. Before (top) and after grid network reduction.

4.2 Coordination opportunities within the hull

Coordination sites are limited by the nature of a grid to vertices and edges and can not occur at the spaces in between. This reduces the potential for coordination in the rectilinear grid network. In contrast, there is no limit on coordination site location in the Euclidean plane. Coordination can be restricted further to occur only at vertices. There is therefore only a finite number of possible coordination sites.

In the rectilinear grid, deviation can occur at any vertex on the existing path, including the preceding node. There is a finite number of deviation points. In the Euclidean plane, routes deviate only at the preceding node.

5 Conclusions

Coordination is a means of reducing distance travelled and fleet size. It involves a consideration of spatial and temporal characteristics so that vehicles arrive at suitable coordination points in the right order or simultaneously. The concept is independent of route construction method and of frame of reference. Different types of coordination effects occur under different circumstances: multiply-routed vehicles, intersections, splittable loads.

Rectilinear grid networks can be reduced in complexity by removing edges that do not contribute to the process of coordination.

References

[1] H.M. Baker, L.S. Franz, J.R. Sweigart, *Coordinated transportation systems: An alternative approach to traditional independent systems*, European Journal of Operational Research, 66 (1993), pp341-352.