The Scheduling of Forest Harvesting with Adjacency Constraints

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Abstract

The forest harvesting problem, FHP, is described. The difference is explained between a strategic model that sets long-term harvesting goals in terms of the total area to be cut each year, and a tactical model that produces a short-term schedule of actual blocks. Reasons are presented for the desirability of an integrated model, embracing both strategic and tactical decisions, which is capable of optimisation. A brief outline of one such model is given. A column generation algorithm is then developed to solve the relaxed linear program formulation. Finally constraint branching techniques are utilised to obtain the desired optimal solution to the integrated model.

1 The Forest Harvesting Problem

Large plantations of exotic timber represent an important asset of international significance. The problem is to find an optimal harvesting plan taking into account all practical considerations concerning both economic and site-specific constraints. The task of formulating this problem as a mathematical model and then constructing an algorithm capable of finding a solution is called the Forest Harvest Problem, FHP.

A commercial production forest is subdivided into units called *blocks*. Each block must be harvested entirely during one felling operation. The trees in a forest are not all the same, and do not grow at the same rate. However, there is a strong tendency for trees in the same block, which are planted at the same time and get the same management regime, to grow at a uniform rate. A *croptype* is a set of trees of the same species and seedling type, which obtain the same management. Trees of the same croptype yield a predictable output per hectare depending on the year of establishment and year of harvest.

A *harvesting model* is a mathematical representation describing the salient aspects of the forest situation and the likely results following various harvesting decisions. These harvesting decisions can be categorised into two types. A *strategic decision* is one pertaining to forest-wide data. For example,

"In 1998 harvest 188.4 hectares of pruned radiata established in 1971",

is a strategic decision. The timber may be cut from any block in the forest provided it is of the designated croptype. The *strategic horizon* refers to the length of time for which strategic decisions are to be made in our solution to FHP. A *tactical decision* is one concerning an actual block or specific part of the forest. For example,

"In 1998 harvest the trees in block 32/5",

is a tactical decision. The trees to be cut must be precisely those located in the stated block. Thus tactical decisions are sensitive to site-specific requirements, whereas strategic decisions are insensitive.

The length of time for which tactical decisions will be made is the *tactical horizon*. Characteristically, a tactical horizon is 2 to 6 years while a strategic horizon is 30 to 100 years. A *strategic plan* is a collection of strategic decisions. A *tactical plan* is a set of tactical decisions. An *integrated plan* contains both strategic and tactical decisions. A strategic model, such as that of Manley et al [3], is commonly written as a linear programme. Unfortunately, strategic models do not generally produce feasible tactical solutions. Tactical models usually contain integer variables. For example, that of Sessions and Sessions [8] formulates FHP as a mixed integer programme and then uses a Monte Carlo integer programming solution process.

Instead of treating the strategic and tactical plans separately, the present model is a single fully integrated model which deals with both harvesting and road construction decisions. A single optimisation process will solve both strategic and tactical sections of the formulation all at once. The main motivation for this approach has been that it offers the best prospect of achieving true optimality. Previous models of this type, such as that of Nelson et al [6], were heuristic in nature.

2 The Model

The main variables are either continuous variables, x_{cet}, used in the strategic part of the model, or binary integer variables, g_{jtn}, used in the tactical part of the model. These are
defined in the following way.

x_{cet} is the number of hectares of croptype *c*, established year *e*, harvested in year *t*.

g_{jtn} determines whether or not the *n*-th harvesting plan, which commences in year *t* and concerns road *j*, will be implemented.

For each of these g_{jtn} variables there is a *road harvest plan* comprising a set of all the harvesting decisions to be taken pertaining to the set of blocks accessed the given road. Here is a typical example.

"On road 17 fell block 2 in year 3, blocks 5 and 6 in year 4 and block 1 in year 5."

Where blocks, such as 3 and 4 in the example, are not included in the road harvest plan it is implied that they will not be harvested during the tactical horizon. Each such road harvest plan is represented by a unique column in the matrix representation of the model. During the solution process many such plans may be constructed, but in the final solution only one road harvest plan will be allowed for each road.

2.1 The objective function

The strategic part of the objective function will contain terms indicating the discounted income associated with the continuous variables x_{cet}. The tactical part of the objective will contain terms indicating the discounted costs associated with the integer variables g_{jtn} . These are combined to represent the present net worth of the forest in relation to the various proposed harvesting plans.

2.2 Strategic constraints

The formulation of strategic constraints has been already well developed, and somewhat standardised. Manley et al [3] present excellent commercial software which takes raw forestry data, formulates it into a linear programme which models the strategic plan, and then solves it. As well as the usual components for such a plan, however, there is a significant possibility of including some non-standard features such as the catchment constraints used by McNaughton [4].

2.3 Tactical constraints

A very great degree of variation occurs here between papers in the literature. The formulation used in the current model is rather similar to the Integrated Resources Planning Model developed by Kirby et al [2], with the important difference that the present model has a column generation structure. The following binary variables are required to model road construction.

r_{jt} determines whether or not road *j* will be operational by year *t*.

Simple road construction constraints, of the form

$$r_{jt} - \sum_{n} g_{jtn} \geq 0,$$

ensure that the necessary roads must be operational before any harvesting is to start. The notorious adjacency constraints are included explicitly, but in an aggregated fashion similar to that advocated by Murray and Church [5].

2.4 Linking constraints

These crucial constraints contain a mixture of strategic real variables and various of the binary tactical variables. So far, the formulation consists of two disjoint parts, as the strategic continuous variables and the tactical binary variables have not jointly appeared in any constraint. Since an integrated model is being constructed, which will be optimised as a single unit, it is essential to introduce a set of constraints that establish the correct relationship between these two principal parts of the formulation.

Constants, a_{cejk}, and sets of harvesting plans, G_{jtk}, are defined for these constraints.

a_{cejk} = the area of croptype *c* established in year *e* located in block *k* on road *j*.

G_{jtk} = all possible harvesting plans in which block *k* on road *j* is felled in year *t*.

One way the formulation of these constraints could be attempted is shown in Equation (1). Here the expression on the left-hand side represents all the timber obtained from the blocks for which felling commences during year *t*.

$$\sum_{j} \sum_{k} \left[a_{cejk} \sum_{G_{jtk}} \left(g_{j\bar{t}n} \right) \right] = x_{cet} \tag{1}$$

Unfortunately, Equation (1) does not represent the harvesting situation accurately in that it assumes every designated block is completely harvested by the end of each year. In practice there are always a few blocks in the process of being harvested for which harvesting will be completed at the start of the new year. Moreover, if implemented this equation causes extreme feasibility difficulties due to excessive tightness in the model. The solution of this problem involves adjusting the formulation of the linking constraints so that they model reality more faithfully. At the end of each year a few of the blocks are likely to be in the process of being felled, with the operation completed
at the start of the new year. New continuous variables, s_{cet}, are defined.

s_{cet} = the area of croptype *c* established in year *e*, intended for felling in year *t*, but which is actually harvested in the following year.

The linking constraints then take the form

$$\sum_{j} \sum_{k} \left[a_{cejk} \sum_{G_{jtk}} \left(g_{j\bar{t}n} \right) \right] + s_{ce(t-1)} = x_{cet} + s_{cet}. \tag{2}$$

In equation 2 the left hand side represents the combined area of all the blocks designated for harvest commencing in year *t*, plus the residue comprising the area remaining to be harvested from the previous year. The right hand side shows the actual total area harvested in year *t*, as the strategic variable x_{cet}, along with the area from the blocks in G_{jtk} which will be left at the end of year *t* and be harvested in year *t*+1.

3 A brief description of the solution algorithm

A column generation algorithm is applied. The work of Weintraub et al [9] is significant in this area. Each column generated represents a unique road harvesting plan pertaining to one of the road segments in the forest. The technique by which this is achieved involves the use of elementary columns. The model contains one elementary column for each block, for each year of the tactical horizon. So each elementary column represents a road harvest plan consisting of one single harvesting decision. The column generating device uses the reduced costs of these elementary columns in order to find attractive combinations which form eligible entering columns. A feature of this non-standard column generation technique is its speed of execution.

During the solution process the LP solving software always treats the problem as just one large relaxed LP. Integer solutions are obtained by a branch and bound process of the constraint branching type, similar to that developed by Desrochers et al [1] and Ryan [7]. In the present application this entails using the sets G_{jtk} in the formulation of branching decisions. For example, a typical branching node consists of a 0-branch

$$\sum_{t \le T} \sum_{Gjtk} g_{j\bar{t}n} = 0,$$

and a 1-branch

$$\sum_{t \le T} \sum_{Gjtk} g_{j\bar{t}n} = 1.$$

Thus the 0-branch prevents the adoption of any road harvest plan in which block k is felled by year T. The 1-branch requires that one of these harvest plans must be chosen, without specifying exactly which of all the many eligible plans this will be. One compelling reason for the use of constraint branching is the dramatic improvement in computational time which results. However, this technique cannot be applied unless the model has been formulated in an appropriate manner. In the present case the sets of road harvest plans, G_{jtk} , have been built into the model for this purpose.

4 Results from a case study

This algorithm has been applied to the Whangapoua Forest at Coromandel, New Zealand. This is a production forest of 7365 hectares which is managed by the Ernslaw One Company. Harvesting of this forest has been modelled with a strategic horizon of 30 years, along with a tactical horizon of 6 years. There are 38 road segments and 145 blocks in the mature part of the forest. A total of 132 pairs of blocks shared adjacency constraints. Prior to the column generation process the matrix representing the model contains 2223 rows and 3380 columns. There are 1290 binary integer variables. As the column generation proceeds, the number of columns and the number of integer variables both increase considerably. If conventional variable branches had been used in the branch and bound process, then no optimisation would be possible, due to the immense combinatorial complexity resulting from this large number of integer variables. However, the constraint branching performs very effectively, delivering an optimal solution at a depth of about 80 nodes on the binary tree. The computational time involved is only about 12 minutes. It is significant that the integer solution obtained in this manner is over 99.95% the value of the objective value obtained at the relaxed LP stage of the solution, prior to any branch and bound.

Thus the development of an integrated model for a large application involving seemingly separate strategic and tactical sections has been vindicated.

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