Optimising Petrol Distribution

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Abstract

Practical aspects of the distribution of petrol across a major city are discussed. Key variables are identified. Practical requirements are formulated into constraints. An objective function represents total cost. This will be minimised. The completed model has a MIP structure. An optimal solution is obtained to the relaxed linear programme. An integer solution is found by constraint branching techniques. Typical performance levels are indicated by reference to trial runs. The chief merit in this application lies in the thoroughness of the model formulation and in the detail of the output data.

1 Introduction to practical aspects of the application

Large petrol companies distribute petrol from a bulk supply depot to a network of about 60 service stations by a fleet of about 10 tankers. Delivery is generally irregular. The primary goal is to meet a fluctuating demand so that no station ever runs out of petrol. Present methods appear manual. Station managers notify the distribution manager of their stock level twice daily. All distribution decisions are made by the distribution manager on a day to day basis using simple rules of thumb such as "always begin by sending the largest truck to the furthest station requiring supply". Such decision making is clearly suboptimal. When the significance of petrol costs in a modern economy are considered, an optimisation model is clearly very desirable. Unfortunately, the cloak and dagger mentality of this industry may inhibit this development.

Containing as it does an embedded traveling salesperson problem, the petrol distribution problem is extremely complex. Refer [1]. Planning needs to involve distribution for several days ahead, if optimality is to be achieved and stock-out avoided. A distinction needs to be made between day-time and night-time delivery and demand. This can be done by regarding each 24 hours as divided into a day shift and a night shift. The model permits a subdivision into 3 or more shifts if this is required. The involvement of several trucks generates much complexity, as the precise schedule for each truck must be accounted for and optimised. This model will represent the entire operation, involving up to a 14 shifts, as a single optimisation.

Some practical aspects of the application have not been addressed in this model. These include distinguishing between the 3 types of fuel delivered, that is regular, premium and diesel.

2 The decision variables and the objective function

Binary integer variables, $x_{i,j,k}$ will represent a journey from station i to station j during shift k. The bulk supply is represented as station 0. The coefficients of these variables in the objective function will represent the cost associated with the time taken by a tanker to make this journey. Other binary variables, v_{ik}, represent a visit to station i during shift k.

Other binary integers, n_k represent the tankers used during shift k. The coefficients of these variables in the objective function will represent the cost resulting from operating a tanker for a 12-hour shift. This is typically a very significant cost.

Three types of continuous variables will be used to record time. The variable $d_{i,k}$ represents the time, in minutes measured from the start of shift k, at which a truck departs from bulk supply heading for station i. The variable $r_{i,k}$ represents the time, in minutes measured from the start of shift k, at which a truck returns directly to bulk supply from station i. Each of these variables takes zero value in the case when such a trip does not occur. The difference between these time variables represents the actual time tankers are on the road. Appropriate coefficients in the objective function represent the associated maintenance and labour costs. A third type of variable $t_{i,k}$ represents the time, in minutes measured from the start of shift k, at which a truck arrives at station i to deliver petrol. There is no cost in the objective function associaterd with these variables.

Two other types of continuous variables are used to record quantities of petrol. The variable $y_{i,j,k}$ represents the amount of petrol in litres carried between stations i and j during shift k. Associated with these variables there is a cost in the objective which represents the extra wear and tear on vehicle maintenance due to this loading. The variable s_{ik} represents the amount of petrol in litres supplied to station i during shift k. There is no cost associated directly with these variables.

3 The constraints

As this is a large and very complex application, a considerable number of constraints are needed to achieve a consistent and detailed model.

There must be no more than 1 truck visiting each station each shift.

The number of trucks leaving a station must equal the number arriving.

If a truck goes directly from station i to station j, then the time it reaches station j must be no earlier than the time it leaves station i plus the time taken to go from i to j.

The load carried from i to j during shift k must be 0 if $x_{ijk} = 0$.

The amount supplied to station$i$, s_{ik}, and the time this station is reached, t_{ik}, must both be 0 if $v_{ik} = 0$.

The load carried to station i minus the amount carried from this station, during shift k, must equal the amount supplied, s_{ik}.

No station must ever stock out. An appropriate safety stock will be set for each station which recognises the stochastic aspects of the situation, and also the actual distance of this station from bulk supply if an emergency delivery were needed. It is sufficient for this constraint to be applied at the time of each delivery, and also at the end of the last shift.

The amount supplied plus existing stock levels, must not exceed the holding capacity of each station.

A non-declining stock constraint ensures that the total amount distributed during each shift is never less that the actual total demand during that shift.

Bounds are placed on the various time variables, [this is the length of a shift, 12 hours], the binary variables [1], the maximum number of trucks available, and the minimum amount of a delivery [typically 2000 litres]. There is a capacity bound on every truck [typically 40000 litres] and on every station.

4 The solution process

The relaxed linear programme is solved using CPLEX. A branch and bound process is then used to obtain an integer solution. Improved integer solutions are obtained by further seaching of the binary decision tree. It is envisaged that in a practical implementation, the algorithm would be run to obtain an optimal distribution plan for several days ahead, preferably for a week. Then each day as stock levels were reported, checks would be made to see that this plan remained valid. If the new data is significantly different from the anticipated demand, then the data files can be updated, the algorithm run again, and a modified plan will result.

5 The branch and bound procedure

The use of constraint branching developed by Desrosiers [2] and Ryan [5] and [6], has proved of major benefit in large mixed integer problems of this type. The present model uses constraint branching to determine which stations need to be visited during the various shifts. At present variable branching is still needed to obtain an integer solution foe all the x_{ijk} variables. However, it is hoped that during further development the use of other constraint branches may allow the elimination of all variable branching.

6 Some typical output and performance levels

The output obtained includes a detailed trip for each truck every shift. Drivers are told how much to collect from bulk supply, and how much to deliver at each station. The actual time they are scheduled to visit each station is given. Summary information for the distribution manager states the total number of tankers needed each shift, and also the overall cost of the distribution process.

The present model is capable of finding a good-quality integer solution to an application involving 30 stations over 4 shifts in about 10 minutes. This is not as good as hoped for. It is anticipated that on-going improvements will significantly shorten this time.

7 Conclusions

The present model has been proven consistent and effective with moderatly sized applications. Further development is planned with an aim of achieving an improved performance level with large applications. For this to succeed, aspects of the formulation may need to be adjusted. A process of constraint aggregation may prove beneficial. Also column generation may be used to reduct the number of variables in the manner of Desrochers et al [2] or McNaughton [4].

The masters dissertation of Hsin-Yee Lin [3] which involved the optimisation of bread delivery, was used to assist the early stages of formulation for the present model. It is hoped that a meaningful dialogue may be opened with a petrol company with a view to implementation in the near future.

References

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