Inventory Control with Overtime and Premium Freight

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Abstract

We consider a manufacturer that must fill stochastic demand. When shortages occur, the manufacturer may choose to meet the unmet demand with overtime production and/or by premium freight shipments. We derive optimal production policies for regular and overtime production and discuss the tradeoffs involved.

1 Introduction

Last year a helicopter crashed outside of Detroit, Michigan; fortunately, no one was seriously injured. The helicopter was carrying automobile parts across the state from a supplier to a manufacturer. Is it ever cost-effective to ship parts by helicopter rather than by truck or rail? Desiring an answer to this question is part of the motivation for our research. Section 1.1 describes the motivation and outline of our current and future research. Section 1.2 discusses related literature and Section 1.3 discusses industry involvement.

1.1 Motivation and Outline

In the past, manufacturers and suppliers have kept high levels of inventory to minimize the likelihood of shortages. However, holding inventory has both direct and indirect costs and during the last two decades many companies have started to implement policies aimed at reducing inventory, or in other words, "going lean" (see, e.g., Liker [3]). With the recent trend towards lean production, inventory levels are so low that the frequency of expedited shipments has increased (according to our contacts at Visteon Automotive Systems in Ypsilanti, Michigan.)

The lean manufacturing paradigm has become popular in industry for one main reason: it is believed to reduce costs. The lean manufacturing philosophy is to reduce costs by continuously improving quality and minimizing inventory. While the successful implementation of lean manufacturing may indeed reduce overall costs, problems with unmet demand may occur with acute reduction of inventory levels. Some of these problems may be remedied by filling the unmet demand using overtime production and/or by building the products later and shipping them by premium freight (e.g., by helicopter) to ensure on time delivery.

It is particularly important for auto-assembly that all parts are at the assembly line on time because shutting down the line is *extremely* expensive. Therefore parts suppliers find themselves making a three-way trade-off between inventory, overtime, and premium freight. In particular, Visteon Automotive Systems faces this exact trade-off when supplying Ford assembly lines and is very interested in this research.

We are studying how to minimize costs in supply chains given that low inventory is desirable and the option exists to meet demand on time with overtime production or by shipping products premium freight. Our objective is to model this system, derive structural results for optimal policies, and answer various questions about the behavior of the system under different circumstances. For example, how does the system respond when inventory holding costs are considered to be very high (as is the case in lean production systems)?

In order to understand this problem, our initial research has been with a single location model. We assume that a single manufacturer has two modes of production: regular and overtime. We also assume that regular production occurs before demand is realized; if there is a shortage after demand is realized, the manufacturer must fill the shortage with either overtime or premium freight. It is important to note that, in our model, demand is always met; in most other inventory models, unmet demand is either backlogged or considered lost. Our initial results show that threshold type policies are optimal for regular and overtime production. Details are given in Section 2.

This paper is organized as follows. Section 2 gives a thorough introduction to our preliminary single location model and results and to what extensions we believe are possible. Section 3 discusses our future research on supply chains, the value of information, and other managerial insights we hope to develop. Finally, Section 4 concludes the paper.

1.2 Related Literature

The seminal paper about threshold policies for single location inventory problems is due to Scarf [6]. In his paper, he shows that under fairly general conditions, the optimal production policy is an (s,S) type policy where it is optimal to produce up to S if the inventory falls to or below s and do nothing if the inventory is greater than s. However, our problem does not meet Scarf's conditions but does closely follow the conditions set forth by Veinott [8]. In his paper, he proves the same results, but allows for a more general cost structure.

The two papers mentioned above both consider a finite time horizon. To tackle the infinite time horizon case, we consider the paper by Zheng [9]. In this paper, the author proves the optimality of an (s,S) type policy over an infinite horizon under conditions similar to Veinott's. Many related papers exist, but to the best of our knowledge none have considered our case with no shortages and with the option of overtime and/or premium freight.

A number of papers have been written on the topics of expediting shipments and overtime production (see Arslan, Ayhan, and Olsen [1] for a very recent review of this literature). Perhaps the most well known paper on expediting is by Fukuda [2]; in this paper he considers product delivery with negotiable lead times, where later deliveries are at a discounted cost. In his paper Fukuda derives optimal policies under the condition that products can be delivered with a normal lead time, or with a one period delay. Moinzadeh and Schmidt [4] discuss emergency orders where different methods of resupply have different lead times. They derive steady-state behavior for an (*S-1,S*) policy and provide sensitivity analysis. However, our assumptions differ from those of both papers listed above in that we assume that all demand must be met on time, and we assume that two options (overtime and premium freight) exist to guarantee timely delivery of goods.

1.3 Company Involvement

The problem of balancing inventory, overtime, premium freight, and information originated during a discussion with the Logistics department at Visteon Automotive Systems. Visteon is a subsidiary of the Ford Motor Corporation that supplies various parts used mainly in the production of Ford automobiles. These parts need to be available at the Ford assembly plants or else production lines will be shut down at an enormous cost.

The Visteon plant in Ypsilanti produces starters for engines. Some of these starters are sent to a Ford plant that produces engines for sport utility vehicles (SUVs). As the demand for such vehicles is high, the engine plant is effectively a bottleneck and therefore needs to produce as many engines as possible. Due to variations in the production process (e.g., machine breakdowns) the number of engines produced per day is not consistent. This makes for variable demand. Therefore, sometimes the demand for starters faced by the Ypsilanti plant exceeds the forecasted value significantly and the managers at the plant must find a way to meet this demand. On the other hand, the typical assembly plant supplied by Visteon is usually very consistent in its demand because it tends to have planned production that results in reasonably accurate forecasts.

In the spirit of lean production, inventory levels at Visteon are kept very low (about half a day’s worth according to our contacts). So when demand is very high, shortages occur. When shortages happen, management must decide whether to run overtime production, wait until the next day and ship the unmet demand by air rather than by ground, or both. Overtime incurs a high setup cost as well as high per unit costs; shipping parts by airplane or helicopter is clearly more expensive than shipping by truck or rail. Both options are expensive, but if the parts are not delivered on time, production lines may have to be shut down and this is an unacceptable alternative. According to our contacts at Visteon, overtime production is common and they ship parts by chartered airplane “often.” In some cases, the parts need only be shipped within the Detroit area; however, instances were mentioned where parts were flown by chartered airplane to Mexico and Japan!

Our current research models the single Visteon facility. We are trying to determine what inventory policies are optimal and when to use overtime and/or premium freight. In real terms, we are trying to determine if half a day’s worth of inventory is enough and whether it is better to keep workers late or to ship parts by helicopter the next morning when necessary. Our future research will look at Visteon as part of the entire Ford supply chain and will consider how advanced information can be used to prevent shortages and reduce costs.

2 Single Location Model

2.1 Model Formulation

Our model consists of a single manufacturer that produces one product. At the beginning of a period, the manufacturer observes the current inventory level. He then chooses the regular time production quantity, which must be enough to make the inventory level non-negative. Then he observes demand. If demand is higher than the current inventory position, the manufacturer must decide how to meet the excess demand by running overtime production and/or by waiting until the next period and expediting the shipment of the unmet demand.

One interesting aspect of this problem is that the manufacturer must make two decisions during each time period. First, he must decide how much to produce during regular production given the current inventory level. Then, after observing demand, he must decide how much to produce during overtime. Note that after the overtime production is run, the starting inventory for the next period is completely determined and any remaining unmet demand must be produced at the beginning of the next period and shipped by premium freight. Our goal is to determine optimal production policies for both regular and overtime production.

This two-decision problem leads to two separate functional equations that depend on each other for the finite horizon case. In the first equation, $f_{rt}(x)$ represents the minimum cost-to-go given that the inventory level before regular production during time t is x. In the second equation, $f_{ot}(x)$ represents the minimum cost-to-go given that the inventory level after demand is observed and before overtime production during time t is x. Note: for a horizon of T periods, we define $f_{r(T+1)}(x) \equiv 0$ for all x. The equations are:

$$f_{rt}(x) = inf_{\{y \geq x+\}} \{K_r d(y-x) + (c_r - c_o)y + E[f_{ot}(y-D)]\}$$

and

$$f_{ot}(x) = inf_{\{y \geq x\}} \{K_o d(y-x) + c_o y + (h-ac_r)y^+ + a(a+c_r)y^- + aAd(y^-) + af_{r(t+1)}(y)\}$$

where $d(x)=1$ if $x > 0$ and 0 otherwise. The per unit costs of regular and overtime production are, respectively, c_r and c_o per unit. The setup costs of regular and overtime production are, respectively, K_r and K_o. The holding cost is h per unit, the cost of premium freight is a per unit, and the setup cost for premium freight is A. The random demand is D and the discount factor is a, with $0 < a < 1$.

The basic assumptions are that:

a) regular production, overtime production and premium freight each incur a non-negative fixed setup cost (K_r, K_o, A ³ 0), with the setup cost for overtime production the largest of the three (K_o ³ K_r, A);

b) regular production, overtime production and premium freight each incur a positive per unit cost (c_r, c_o, $a > 0$) with the per unit cost for overtime production greater than the per unit cost of regular production ($c_o > c_r$) and the discounted per unit cost of premium freight and regular production greater than the per unit cost of overtime production ($a(a + c_r) > c_o$);

c) the manufacturer incurs a per unit holding cost (h) for positive inventory remaining after overtime;

d) every unit of unmet demand is filled either by overtime production or by expedited shipping at the beginning of the next period;

e) each period the demand is independent and identically distributed and follows a logconcave probability distribution (see Rosling [9] for details); and

f) the manufacturer has infinite capacity during regular production and overtime production.

Our goal is to minimize the manufacturer's total expected discounted cost over the finite and infinite time horizons. We also wish to minimize the manufacturer's average cost over the infinite horizon.

We are convinced that threshold type policies are optimal for the general model for both finite and infinite time horizons and are currently in the process of trying to prove this. However, we do have proof that threshold type policies are optimal if we relax some of the assumptions above. First, if we assume that there is no setup cost for regular production and premium freight ($K_r = A = 0$), we can show that the optimal regular production policy is a base-stock policy and the optimal overtime production policy is an (s,S) policy. Details are available in Section 2.2. Second, if we assume that the demand distribution is exponential, we can show that the optimal regular and overtime production policies are (s,S) policies. Finally, we have studied some interesting results in the general case which will be discussed in Section 2.3

2.2 No setup cost for regular production and premium freight ($K_r = A = 0$)

For the finite horizon version of this case, we have proved that threshold policies are optimal for regular production and for overtime production.

Proposition: The optimal regular production policy is a base-stock policy and the optimal overtime production policy is an (s_o, 0) policy with

$$s_0 = \frac{K_0}{c_0 - a(a+c)}.$$

Thus, during regular production time, an optimal base-stock level exists and the optimal policy is to produce up to the base-stock level whenever the inventory level falls below the base-stock level. During overtime, if the inventory level is positive, do nothing. If the inventory level is negative but not less than s_o, do nothing now but prepare to ship the unmet demand by premium freight at the beginning of the next period. Finally, if the inventory level is less than s_o, produce enough during overtime to bring the inventory level up to *0*.

Note that s_o is negative by assumption b). Also notice that as the setup cost for overtime, K_o, increases, the threshold becomes more negative. This makes sense intuitively since the manufacturer will want to avoid overtime when the setup cost for overtime production is high. Also notice as the denominator (the difference between the per unit costs of premium freight and overtime) decreases, the threshold becomes more negative. This makes sense as well, since if the two per unit costs are close, the manufacturer will choose the premium freight option to avoid the setup cost.

The proof of our proposition proceeds as follows. First, we show that the optimal overtime production policy is the (s_o, *0*) policy discussed above; then, we show that the optimal regular production policy is a base-stock policy by an inductive argument. To show the former, we show that when the overtime decision is made, the choice must be for overtime production or premium freight, but not a combination of both. Using this fact and comparing costs, we prove that the threshold is s_o.

To prove the latter, we start at the end of the horizon (time *T*) and show that a base-stock policy is optimal for regular production. We then inductively prove that the same base-stock policy is optimal for every time period. The most interesting part of the proof is that we must show that *G(y)*, our single period cost function which includes the costs of production, holding, and premium freight, has a unique minimum. To prove that *G(y)* has a unique minimum, we require that our demand distribution is logconcave and hence assumption e). Note, however, that most demand distributions commonly used in inventory literature are logconcave, including the Normal, Uniform, Exponential, Gamma, Beta, and Poisson distributions.

We show that *G(y)* has a unique minimum by considering its derivative, *G¢(y)*, where

$$G¢(y) = c_r - c_o + (h+aa)F(y) + (c_o - a(a+c))F(y-s_o)$$

and $F(\cdot)$ is the cumulative distribution of demand. We show that *G¢(y)* is negative on the left and postitive on the right and that it has exactly one zero. Thus, *G(y)* has a unique minimum.

2.3 General case results

We are convinced that in the general case, both the optimal regular production policy and the optimal overtime production policy will be (*s,S*) policies. We are modeling our proofs on the paper by Veinott [8] for the finite horizon case and on the paper by Zheng [9] for the infinite horizon case. In both cases, we are trying to show that the best overtime production policy is (*s,S*) first and then use the results from the two papers to show that the best regular production policy also (*s,S*). The problem is much more interesting in this case. (And, of course, much more difficult to prove!)

The main difficulty occurs when we calculate s_o, the threshold between overtime production and premium freight. In the case where there is no setup cost for regular production, it is easy to show that if overtime is used, the inventory level will always be brought up to *0*. However, if there is a setup cost for regular production, it may be optimal during overtime to bring the inventory level up to some positive value to avoid the regular production setup cost the next period. Intuitively, if the overtime production option is chosen, the overtime production setup cost becomes a sunk cost; in addition, if the per unit cost of overtime production is relatively close to the per unit cost of regular production, it makes sense to produce up to a positive inventory level during overtime and avoid regular production the next period.

Consequently, the value of s_o may not be stationary which complicates both proofs. In this case, the parameters for the opimal overtime policy may depend on the time period *t*; in other words, the overtime production theshold will be s_{ot} and the optimal inventory level to produce up to during overtime production will be S_{ot}. We can still show that $G_t(y)$ has a unique minimum (although it may now depend on *t*), but for the finite horizon case, the non-stationarity complicates the inductive proof. For the infinite horizon case, we believe that these values will be stationary, but our proof is not yet complete.

3 Future Research

When we have proven that threshold policies are optimal for the single location problem, our next step will be to investigate whether these policies hold for the entire supply chain. Our original problem came from Visteon Automotive Systems, which is an integral part of the Ford supply chain, and thus we are very interested in results for the multi-echelon problem. Another area we plan to investigate is the value of information. We believe that if the Visteon plant could get real-time data from the assembly plants it supplies, inventory shortages and over-production could be drastically reduced and we hope to quantify the savings involved.

3.1 Supply Chain

In this section we discuss three types of supply chains, namely, centralized, cooperative, and competitive. Centralized and cooperative supply chains have a single objective while competitive supply chains have multiple players each with his or her own objective. While the global objective is the same in centralized and cooperative supply chains the information available at each stage is not. These differences mean that different research questions are of interest in each of the three types of supply chain and we therefore discuss them individually.

Our first supply chain model will consist of two stages that are all controlled by a single controller whose goal is to minimize total system cost. We will assume that the controller has perfect information from each stage and that at least one of the stages has the possibility of overtime and premium freight shipments. Our intuition is that the total supply chain savings using an optimal policy for the entire system will be significant, and we expect to apply real world data from the automotive industry to verify our results.

The centralized model assumed that global information was available; this is clearly less than realistic. Therefore, our second supply chain model will consist of two stages that belong to the same corporation, but have only local information and may be operated independently. We wish to know if a set of locally optimal policies exists that will induce global optimality achieved by the centralized model.

Finally, our third supply chain model will consist of two stages that are not only independent, but also competitive. At both stages, the managers will be concerned only with minimizing their own costs. It seems unlikely that the results for the two models mentioned above will apply directly to this situation, but it may be possible to achieve system-wide optimality with appropriate contract structures or other measures. In other words, it may be possible to design contracts that result in both competitors working towards system-wide optimality.

3.2 Value of Information

One of the most intriguing concepts in supply chain management is the value of information. We will investigate the role of information in all of the models discussed above, and we hope to gain insight into the actual savings that good information can provide.

In the single location model, we expect the cost to decrease as better information becomes available. The assumption in our current model is that the manufacturer knows only the demand distribution and must react with overtime or premium freight after the demand is realized. If the demand is highly variable, the costs associated with filling unmet demand will be extremely high. However, what if the demand variability can be smoothed by increased information flow from downstream, such as inventory position or a rolling forecast of demand? In this case, we expect to find significant savings for the manufacturer.

In the supply chain model, we believe that information will also be valuable. In the centralized model we will assume that a central controller knows all the information about the entire system. The controller should be able to coordinate production along the process smoothly and make sure that each stage has accurate information about what is going on downstream. The total supply chain cost will be minimized in this model.

However, things will not be so simple in the cases where each stage of the supply chain is independent and cooperative or where each stage of the supply chain is competitive. In the former, the challenge is to find a way to use information at the local level to achieve global optimality. In the latter, the challenge is to find a way to share the savings from minimizing total supply chain cost among all competitive stages such that at each stage, the benefit of shared savings outweighs the benefit of achieving local optimality.

4 Conclusion

The current trend in much of industry is towards lean manufacturing. This trend has created new challenges in the field of inventory control. One such challenge is how to meet stochastic demand consistently while holding relatively low levels of inventory. In our problem, we consider two alternatives to deal with shortages: overtime production and premium freight shipments.

We have proven that threshold policies are optimal for relaxations of our general single location model. Our current research focuses on proving the optimality of threshold policies for the general single location model. In the future, we will study our problem in the supply chain context and we will investigate the value of information.

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