Optimal Offer Stacks for Hydroelectic Generators in Electricity Markets

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Abstract

We look to extend some results on offering strategies in an electricity market to the case of a hydroelectric generator, whose costs are dominated by the marginal cost of using water now instead of having it available for later use. In this case, strict concavity of the value of water function can simplify the problem. We give a number of results demonstrating that this strict concavity is inherited when using a dynamic programming method, stepping back from a time horizon. We consider possible extensions to a generation company controlling a number of dams on a river chain.

1 Introduction

We consider the problem facing hydroelectric generators, selling electricity into wholesale electricity markets and attempting to maximise returns. In recent years various forms of these markets have emerged throughout the world. We consider markets that have a dispatch and pricing mechanism similar to that now implemented in the United Kingdom, Australia and New Zealand, where each generator produces an *offer stack*, consisting of a finite set of price-quantity pairs (*tranches*), indicating that they are willing to produce those quantities at the corresponding prices. The number of tranches that can be offered in any offer period depends on the market. (In New Zealand each generator is restricted to five tranches in each half-hour period). The prices in these markets are determined by a central authority (M-Co in New Zealand) that clears demand by systematically dispatching the cheapest supply offers until demand is met (allowing for transmission network losses and constraints). The marginal cost of supply at any node of the transmission network (i.e. the shadow price of the energy balance constraint) then defines the spot price of energy at this node.

In this paper we choose to ignore contracts and deal with a model in which the generators are risk neutral and seek an offer stack in each dispatch period that will maximise their expected revenue minus expected costs. For simplicity we shall begin by restricting attention to a generator offering a single offer stack at a single node of the transmission network, and later consider extension to a series of generators on the same river system.

We begin by summarising some results for a generator in such a market, and then focus on some results specific to hydroelectric generators. Neame et al. [3] deals with the case where the generator is small enough to have a negligible effect on the clearing price in the market. A more general case is considered by Philpott et al. in [4], where the generator's offers do affect the market price, but competitors' actions are assumed to be independent of the policy of the generator we have in mind. They represent the uncertainties in competitor behaviour and demand by a single *market distribution function* $\psi(q,p)$, defined to be the probability of a single-tranche offer (q,p) not being fully dispatched. This can be used to derive optimality conditions for offer stacks in this price-making context. The most general model would include competitors' responses to the generator's actions; a game theory problem that is very difficult to solve.

In the price-taking model that we begin by considering, the probability $\psi(q,p)$ is a function of p only. (It is the probability that the clearing price is below p. For simplicity of exposition we restrict attention in this paper to cases where the price distribution in any offer period has a known density function with bounded support.)

Neame et al. [3] considered the general case of a price-taking generator with an arbitrary cost function, offering a finite number of tranches into an electricity spot market and provided optimality conditions. Here, we begin by analysing the case of a hydroelectric generator in such a framework, and then consider extensions to the case of a hydroelectric generator in a price-making framework, and of multiple generators on the same river chain. We note that strict convexity of the cost function $c\,(q)$ can make our calculations easier.

In particular, in the hydroeclectric case, the cost of production is dominated by the opportunity cost of no longer having the water used available. Hence, for a given current state (water level) x in time period t, and a value-to-go function for the next time period $V_t\,(\cdot)$, we can give the cost as

$$c_t\,(x,q) = V_{t+1}\,(x) - V_{t+1}\,(x-q)\,. \tag{1}$$

For much of this paper we consider a single period with x given, and hence drop x from the analysis, and refer to $c\,(q)$. Note that we assume that a quantity of water x is converted to a quantity of power q in linear fashion, so in the following we consider x to be measured in MWh. In reality, the efficiency of the electricity production depends on both x (via the head, or pressure, generated) and q (as turbines run most efficiently at certain levels) — we neglect these issues for now.

The model which we have in mind is for relatively short-term planning of water usage. Hence, we focus on the price p as being the major stochasticity in the problem, and take fairly crude models for inflow — firstly assuming that there is no inflow, and then taking a deterministic value. This is reasonable in the short-term, as we can assume that the generator has some knowledge of how much water is upstream. Over the longer term, the amount of rainfall and/or snow melt would become significant. We make a brief comment on stochastic inflows, but we largely do not model this; instead we assume that we take T time periods and that there is a function available for the value of water at the end of period T. We wish to explore the structure of the problems at each time period $t = 1, \ldots, T$.

We show that a dynamic programming model with a strictly concave value of water function at the time horizon, leads to strict convexity of the cost function being inherited for all previous time periods.

To begin with, we take the case where the generator offers in a continuous function $q(p)$. Define $V_t(x)$ as the value (in expectation) of x units of potential production in time period t, for $t = 1, \ldots, T$ and $x \in [0, x_{Max}]$, and let the dam have maximum capacity x_{Max}. We extend this to all real x by letting

$$V_t(x) = \begin{cases} -\infty & , \quad x < 0 \\ V_t(x_{Max}) & , \quad x > x_{Max}. \end{cases}$$

2 Offer Stacks

It is convenient to model an offer stack as a continuous curve $\mathbf{s} = \{(q(t), p(t)), 0 \le t \le T\}$, in which the components $q(t)$ and $p(t)$ are monotonic increasing piecewise differentiable functions of t. Here $q(t)$ traces the quantity component of the offer curve and $p(t)$ traces the price component. This definition encompasses the case where the generator offers a continuous supply function $q(p)$, as well as the case where there are a fixed number of tranches — in this case the curve $\mathbf{s}$ will consist of horizontal and vertical sections. Let $f(p)$ denote the probability density function of spot price. We assume f has bounded support giving a bound p_M on the spot price, so that $p(t) \le p_M$, $0 \le t \le T$. Furthermore, we assume that $q(0) = 0$, $p(0) = p_0 = \inf\{p \mid f(p) > 0\}$, and that $q(t) \le q_M$, $0 \le t \le T$, where q_M is the generation capacity of the generator. We also assume that $q(T) = q_M$, and $p(T) = p_M$, which includes the case where the final section of the curve is vertical at q_M.

The single period return can then be modelled as $qp - c(q)$. We are interested in maximising the expected return obtained by choosing a particular stack $\mathbf{s}$ to offer into the market. Then the expected return from offering $\mathbf{s}$ is

$$F = \int_{t=0}^{T} [q(t)p(t) - c(q(t))] f(p(t)) p'(t) dt.$$

If the generator can offer a quantity that is a continuous function $q(p)$ of the price p then the expected return is

$$F = \int_{p=0}^{p_M} [q(p)p - c(q(p))] f(p) dp.$$

Note that if $f(p) > 0$ for all $p \in [p_0, p_M]$, then F is maximized when

$$\frac{\partial (qp - c(q))}{\partial q} = 0,$$

giving $p-c'(q)=0$. Hence the optimal strategy for the generator is to offer a curve $q(p)$ with the property that the price of each offered amount is its marginal cost, provided that this is non-decreasing. This problem could then be solved at each time period in a dynamic programming model, iterating backwards from the time horizon.

However, the marginal cost function $c'(q)$ may decrease over part of the range, and furthermore, the generator cannot offer in an arbitrary continuous function. Instead they may choose no more than m tranches.

3 Optimality Conditions

In the case really facing such generators, $\mathbf{s}$ is a step function that changes direction at points $(0,p_1)$, (q_1,p_1), (q_1,p_2), $\ldots$, (q_m,p_m). If $p_m < p_M$ then there is a vertical section from (q_m,p_m) to (q_m,p_M). The expected return is now

$$F(p_1,\ldots,p_m,q_1,\ldots,q_m)=\sum_{i=1}^{m}\int_{p_i}^{p_{i+1}}\left[pq_i-c(q_i)\right]f(p)dp, \tag{2}$$

where for convenience we identify p_{m+1} and p_M. The problem of computing an optimal offer stack with m or fewer tranches is now the following nonlinear programming problem.

$$\begin{aligned}
\bar{P}(m) \quad &: \quad \max F(p_1,\ldots,p_m,q_1,\ldots,q_m) \\
\text{such that } p_i &\le p_{i+1},\ i=1,\ldots,m-1, && (3)\\
q_i &\le q_{i+1},\ i=1,\ldots,m-1, && (4)\\
0 &\le p_i \le p_M, i=1,\ldots,m, && (5)\\
0 &\le q_i \le q_M,\ i=1,\ldots,m. && (6)
\end{aligned}$$

We begin by considering the case of $c(\cdot)$ nonsmooth, but differentiable almost everywhere. In Neame et al. [3], we show that the following are necessary for a stack to be locally optimal:

$$p_i=\frac{c(q_i)-c(q_{i-1})}{q_i-q_{i-1}},\ \text{for all } i=2,\ldots,m, \tag{7}$$

and

$$c'_+(q_i)\ge\frac{\int_{p_i}^{p_{i+1}}pf(p)dp}{\int_{p_i}^{p_{i+1}}f(p)dp}\ge c'_-(q_i)\ \text{ for all } i=1,\ldots,m-1, \tag{8}$$

where $c'_-(q_i)$ and $c'_+(q_i)$ are the left and right derivatives of c at q_i. In the case where $c(\cdot)$ is smooth, (8) becomes

$$c'(q_i)=\frac{\int_{p_i}^{p_{i+1}}pf(p)dp}{\int_{p_i}^{p_{i+1}}f(p)dp}\ \text{for all } i=1,\ldots,m-1.$$

This condition says that the conditional expected price along the vertical section is equal to the marginal cost.

In Neame et al. [3], we also outline second order conditions for optimality in the price-taking case. Anderson and Xu [1] give second order conditions in the price-making case.

4 Optima with Fewer Tranches

We note that it is possible that all optimal solutions to $\bar{P}(m)$ contain fewer than m tranches. In this case, finding all solutions with m tranches that satisfy the first and second order conditions would not be sufficient to be certain of optimality; we would need to check for solutions with less than m tranches.

We now consider the optimisation problem of finding a solution with exactly m tranches:

$$\begin{aligned} P(m) \quad &: \quad \sup F(p_1, \ldots, p_m, q_1, \ldots, q_m) \\ \text{subject to } p_i \quad &< \quad p_{i+1},\ i = 1, \ldots, m-1, \\ q_i \quad &< \quad q_{i+1},\ i = 1, \ldots, m-1, \\ p_0 \quad &\leq \quad p_i < p_M, i = 1, \ldots, m, \\ 0 \quad &\leq \quad q_i \leq q_M,\ i = 1, \ldots, m. \end{aligned}$$

Note that $P(1)$ is identical to the problem $\bar{P}(1)$. However for $m > 1$, $\bar{P}(m)$ involves maximising a function over an open set, so there may not be a solution. In particular, all optimal solutions to $\bar{P}(m)$ might contain fewer than m tranches. In this case a sequence of feasible solutions for $P(m)$ can be constructed that converges to an optimal solution to $\bar{P}(m)$, but never attains the optimal value.

We now show that in the cases where the cost function c is either strictly convex, or piecewise linear (with $n \leq m$ linear sections) and convex, we do not need to consider solutions with fewer than m tranches to solve $\bar{P}(m)$.

First observe that any feasible solution to $\bar{P}(m)$ which is not feasible for $P(m)$ can be expressed as an equivalent stack of fewer than m tranches, that is, for all p the quantity offered $q(p)$ is the same for each stack. If $q_i = q_{i+1}$ then $\{p_1, \ldots, p_i, p_{i+2}, \ldots, p_m, q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_m\}$ yields an equivalent stack with $m-1$ tranches. Similarly, if $p_i = p_{i+1}$, then $\{p_1, \ldots, p_i, p_{i+2}, \ldots, p_m, q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_m\}$ yields an equivalent stack. If $p_m = p_M$, then by the definition of p_M the final tranche has zero probability of being dispatched, so $\{p_1, \ldots, p_{m-1}, q_1, \ldots, q_{m-1}\}$ is an equivalent stack with $m-1$ tranches. This statement can be rewritten as the following lemma.

Lemma 4.1 *Any solution that is feasible for $\bar{P}(m)$ but not feasible for $\bar{P}(m-1)$ is feasible for $P(m)$.*

Lemma 4.2 *If $c(q)$ is strictly convex, and $f > 0$ almost everywhere on $[p_0, p_M]$, then the optimal solution to $\bar{P}(m+1)$ yields a strictly higher return than the solution to $\bar{P}(m)$.*

Proof. Consider an optimal solution S to $\bar{P}(m)$. Assume for the moment that it is feasible for $P(m)$, and choose one horizontal section of the stack arbitrarily. By (7),

$$p_i = \frac{c(q_i) - c(q_{i-1})}{q_i - q_{i-1}}.$$

For any $\bar{q} \in (q_{i-1}, q_i)$, consider

$$\bar{p} = \frac{c(q_i) - c(\bar{q})}{q_i - \bar{q}}. \tag{9}$$

By the strict convexity of c, $p_i < \bar{p} < p_{i+1}$, so

$$\bar{S} = \{p_1, \ldots, p_i, \bar{p}, p_{i+1}, \ldots, p_m, q_1, \ldots, q_{i-1}, \bar{q}, q_i, \ldots, q_m\}$$

is feasible for $P(m+1)$. The difference in total return between the two stacks,

$$\begin{aligned} F(\bar{S}) - F(S) &= \int_{p_i}^{\bar{p}} (\bar{q}p - c(\bar{q})) f(p)\,dp - \int_{p_i}^{\bar{p}} (q_i p - c(q_i)) f(p)\,dp \\ &= \int_{p_i}^{\bar{p}} (p(\bar{q} - q_i) + c(q_i) - c(\bar{q})) f(p)\,dp \\ &= (q_i - \bar{q}) \int_{p_i}^{\bar{p}} (\bar{p} - p) f(p)\,dp, \text{ from (9)} \\ &> 0, \text{ by the previous assumption.} \end{aligned}$$

Hence $\bar{S}$ is strictly better than the best possible stack using m or fewer tranches, so the optimal solution to $\bar{P}(m+1)$ is strictly better than the optimal solution to $\bar{P}(m)$.

Now recall the assumption that S was feasible for $P(m)$. We show this is true by induction on m. When $m = 1$, $\bar{P}(m)$ and $P(m)$ are identical, so obviously any optimal solution to $\bar{P}(m)$ must be feasible for $P(m)$. For $m > 1$, suppose S is feasible for $P(m-1)$. Now by Lemma 4.1, any solution that is feasible for $\bar{P}(m)$ but not feasible for $\bar{P}(m-1)$ must be feasible for $P(m)$. By the argument in the previous paragraph, the optimal solution for $\bar{P}(m)$ has a strictly greater value than that of $\bar{P}(m-1)$ and so it cannot be feasible for $\bar{P}(m-1)$. Thus it must be feasible for $P(m)$. ∎

5 Convexity and Hydroelectric Models

We now consider some results that suggest that the conditions in Lemma 4.2 are likely to be satisfied in the hydroelectric case. In particular, if we choose a strictly concave value of water function at the time horizon, then the strict concavity is inherited at previous time periods. Note that, by (1), strict concavity of the value of water function corresponds to strict convexity of the cost function.

Lemma 5.1 *Assume that for any* $x < x_{Max}$ *there exists a price* $\tilde{p}(x)$ *such that* $\int_{p_{Min}}^{\tilde{p}(x)} f(p)\,dp > 0$ *and* $V_t'(x) \geq \tilde{p}(x)$. *Suppose* $V_{t+1}(x)$ *is strictly concave on* $[0, x_{Max}]$, *there are no inflows and the generator offers in a continuous stack* $q(p)$. *Then* $V_t(x)$ *is strictly concave on* $[0, x_{Max}]$

Proof. For a given p, let

$$W_t(p, x) = \max_{q(p,x)} \{V_{t+1}(x - q) + qp\}.$$

For a given p and x, there are three cases to check for the optimal q. Firstly, if there exists $q \in [0, x]$ such that $p = -V_{t+1}'(x - q)$, then this is the optimal q. In this case, we label $\tilde{x} = x - q$. Secondly, if $-V_{t+1}'(0) < p$, then $q = x$ and thirdly if $-V_{t+1}'(x) > p$, then $q = 0$. Hence $W_t(p, x) = \max\{xp, V_{t+1}(\tilde{x}) + (x - \tilde{x})p, V_{t+1}(x)\}$. Note that for a given p, $W_t(p, x)$ is concave in x as it is the maximum of three concave functions, but may not be strictly concave in x (see Hiriart-Urruty and Lemaréchal [2] Proposition I.2.1.2).

If we have a continuous distribution for the price $f(p)$ then the expectation across all possible prices p is

$$\begin{aligned} V_t(x) &= \max_{q(p)} \left\{ \int_{p_{Min}}^{p_{Max}} (V_{t+1}(x - q) + qp) f(p)\,dp \right\} \\ &= \int_{p_{Min}}^{p_{Max}} W_t(p, x) f(p)\,dp. \end{aligned}$$

Now we consider whether V_t inherits strict concavity, by considering the functions $W_t(p, x)$. Under the technical assumption that for any $x < x_{Max}$ there exists a price $\tilde{p}(x)$ such that $\int_{p_{Min}}^{\tilde{p}(x)} f(p)\,dp > 0$ and $V_t'(x) \geq \tilde{p}(x)$, $q(p, x) = 0$ for all $p \in [p_{Min}, \tilde{p}(x)]$, and hence $W_t(p, x) = V_{t+1}(x)$. Thus the function

$$\int_{p_{Min}}^{\tilde{p}(x)} W_t(p, x) f(p)\,dp = \int_{p_{Min}}^{\tilde{p}(x)} V_{t+1}(x) f(p)\,dp$$

which is strictly concave.

Now

$$V_t(x) = \int_{p_{Min}}^{\tilde{p}(x)} W_t(p, x) f(p)\,dp + \int_{\tilde{p}(x)}^{p_{Max}} W_t(p, x) f(p)\,dp,$$

which is the sum of a strictly concave function and an infinite sum of concave functions, and hence it is strictly concave (see Hiriart-Urruty and Lemaréchal [2] Proposition I.2.1.1). ∎

We now look at the case where the generator offers in a finite number of tranches.

Lemma 5.2 *Suppose* $V_{t+1}(x)$ *is strictly concave on* $[0, x_{Max}]$, *there are no inflows, and the generator offers in an offer stack* $p_1, \ldots, p_m, q_1, \ldots, q_m$. *Then* $V_t(x)$ *is strictly concave on* $[0, x_{Max}]$.

Proof. We consider the optimal offer stack as a (non-continuous) function $\hat{q}(p)$, and redefine

$$W_t(p, x) = \{V_{t+1}(x - \hat{q}(p)) + \hat{q}(p)p\}.$$

In order to satisfy the first order optimality conditions, $p_1 > p_{Min}$ and $\int_{p_{Min}}^{p_1} f(p)\,dp > 0$. Thus for all $p \in [p_{Min}, p_1]$, $\hat{q}(p) = 0$ and thus $V_{t+1}(x - \hat{q}) + \hat{q}p = V_{t+1}(x)$. Hence $\int_{p_{Min}}^{p_1} W_t(p, x) f(p)\,dp$ is a strictly concave function, and by a similar argument to that used in Lemma 5.1, $V_t(x)$ is strictly concave. ∎

Next we consider the situation of a deterministic inflow of size w. Note that this means that the reservoir capacity x_{Max} is now important, since if the quantity of water at time $t + 1$ is greater than x_{Max} then water is spilt, so if that water is not used in period t, it has zero value. Hence if $w > 0$, $V_t(x)$ will be linear over the range $[x_{Max} - w, x_{Max}]$, and thus no longer strictly concave over $[0, x_{Max}]$.

However, in the case of stochastic inflows, if we make the assumption that there is a finite probability of 0 inflow, then $V_t(x)$ inherits concavity over $[0, x_{Max}]$. Note that 0 inflow does not mean that absolutely no water is flowing down the river, since evaporation can be a significant factor.

Lemma 5.3 *If $V_{t+1}(x)$ is strictly concave on $[0, x_{Max}]$, and the inflow is a random variable ω with $\Pr(\omega = 0) > 0$, then $V_t(x)$ is strictly concave on $[0, x_{Max}]$ if the generator offers in either finitely many tranches or a continuous function $q(p)$.*

Proof. Let the value of water $V_t(x) = \int U(x, \omega)\,d\omega$, that is an expectation over the values of ω. For all ω, $U(x, \omega)$ is concave in x, and $U(x, 0)$ is strictly concave, from Lemmas 5.1 and 5.2. Hence $V_t(x)$ is strictly concave if $\Pr(\omega = 0) > 0$. ∎

6 Extensions

We note that a continuous function of the amount of water in the dam, x, can rapidly become difficult to model. Hence, we could consider a finite number of states (water levels) for the dam. However, as is common with dynamic programming models, we come up against the so-called "curse of dimensionality". We are particularly interested in the case where a single company owns a number of hydroelectric dams on the same river system.

Prior to the implementation of the electricity market, hydroelectric generators needed to allow for uncertainty in future demand, see for example Philpott et al. [8]. Under the market, there is uncertainty in future prices; arguably more so than in future demand. In practice, substantial spikes are seen in prices. A price-taking hydroelectric generator would find it highly desirable to have water available in all of their dams in these periods. The only way to be certain to have water in dams down the river is to have offered in production at price 0 upstream in previous periods, but this may involve foregoing profit in those periods. Hence, the problem becomes a complicated multiple-stage stochastic programming problem, where we wish to hedge against the risk of not having water available when prices are high.

One question of interest is — "what should a value function look like for a number of dams?" Evidently, there is extra value in having more water at the first dam, as that water can be re-used a number of times on its way down the river. However, there are also added constraints from having the succession of dams — each one should try to avoid spilling water. One approach we consider is to collapse the multiple dam model down to a single dam, in order to gain bounds on this value function.

Indeed, a preliminary question is "what should the value of water function look like for one dam?" We have been investigating some simple dynamic programming models for hydroelectric dams, with relatively few possible states. We considered independent prices (independent from each other and from the inflow), and noted that the choice of value of water function at the time horizon is largely insignificant after iterating back several time periods. Indeed, for a wide range of "sensible" choices of terminal value of water functions and probabilities of inflows, the process is a contraction. That is, the value of water function tends to a stable equilibrium. This function is strictly concave (over the discrete dam levels considered). Indeed, with deterministic inflow at regular periods, the value of water tends to a stable cycle of functions (obviously, the value of a full dam is greater the more periods there are before the next inflow arrives).

We have extended this model to one with two dams in succession, where the water used (or spilled) by the first dam provides all of the inflow for the second. In this case, the value of water lies between that of a dam with capacity equal to the sum of the capacities of the two dams, and that of a dam with capacity equal to twice the capacity of the first dam plus the capacity of the second dam. This is, in some respects, an obvious result since the water from the first dam could be used twice, but occasionally the capacity constraints of the individual dams will impinge on the solution. We hope further investigation of this case will yield better approximations of the combined value of water function for multiple dams.

As of October 2000, the New Zealand Electricity Market rules allow for some companies owning a number of hydroelectric generators on the same river chain to have the right to "block dispatch" these generators. This means that each individual generator can offer a stack into the market, but after the market administrator assigns a provisional dispatch to each generator, the company can make some readjustments. That is, within the constraint that the total supply from its generators matches the total assigned to them by the market, the company has the freedom to determine how much supply comes from each individual generator. This condition was apparently added to the market to ease the management of water management within such river chains. However, removing the right to block dispatch has been recently considered by the market rules committee [7]. This raises two questions — "how can water flow through the river chain be managed without block dispatch?" and "what is the right to block dispatch worth to a company?" We hope to begin to address these issues with these models.

We have largely considered the case where the price process can be considered as independent from period to period. Pritchard and Zakeri [5] have extended this to Markov prices (that is, where the likelihood of a price being in a "high band", for example, is dependent on the price in the previous period). This idea appears to match the observed prices in the New Zealand market more closely than an assumption of independent prices. However, their method would rapidly run into problems with the curse of dimensionality in the case with a series of hydroelectric dams.

Another obvious extension is to the price-making case. Many of the hydroelectric generation companies in New Zealand are large enough that they may be able to affect the clearing price of electricity. This greatly complicates the analysis, but substantial work has been done on the question of optimality conditions in such a situation, see [4]. Some of the outstanding issues to be resolved include finding good models of the market distribution function $\psi(q,p)$, and efficient techniques for finding offer stacks satisfying the optimality conditions.

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