Flight Schedule Optimisation for Air New Zealand's International Fleet

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Abstract

Air New Zealand had done very little work in the area of aircraft scheduling using optimisation techniques until last year. A basic optimisation model was developed to allocate flights and maintenance to aircraft over a period of time. This project implements several extensions to this model. Departure time windows were incorporated to allow flexibility within the flight schedule. Each of the individual sectors were also allocated a priority. These priorities were then used to reduce the number of aircraft in the fleet. A linear programming (LP) relaxation and branch and bound approach was used to solve this model as a generalised set-partitioning problem. This approach led to the efficient construction of a good quality legal solution of the Boeing 747-400 fleet.

1 Introduction

The airline industry is a perfect example of where scheduling plays an important role. During the past decades, air transportation has undergone extensive growth. This development has meant an increase in the range of practical problems associated with the business that need to be solved. Almost every aspect of an airlines operation can be considered a scheduling problem. Aircraft, crew, passengers and sundry items must be organised into an efficient operation on a daily basis. Therefore the potential benefits for using optimisation methods to produce optimal or near-optimal schedules is significant. It is these financial motives that are driving the airline industry to make use of optimisation in all areas of operation.

1.1 Air New Zealand

Air New Zealand is the major national and international airline based in New Zealand. Like many other airlines Air New Zealand has a large number of resources to manage. The company must organise the aircraft and then make sure that they are adequately staffed with pilots and flight attendants.

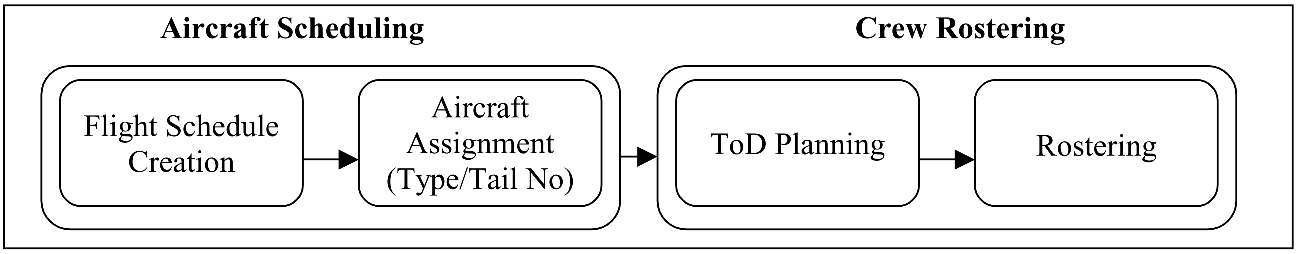
In today's highly competitive market Air New Zealand must efficiently utilise and manage these resources to provide a satisfactory level of customer service or risk losing valuable business.

1.2 Airline Scheduling Problems

The airline scheduling problem consists of several sub-problems that may interact with each other to varying degrees. This results in a large and complex overall problem. Day [2] describes the fundamental problem common to every airline as the “choice of schedule, scheduling of aircraft and the subsequent scheduling of aircrew to those aircraft”.

Attempting to solve the entire problem of aircraft scheduling and aircrew scheduling as a single global problem is impractical. This is why many airlines divide the problem into smaller sub-problems of a more practical nature.

Figure 1 gives an overview of the airline scheduling problem. Usually the aircraft are scheduled first with little or no regard for crew considerations. Crews are then scheduled with the input being the fixed aircraft schedule.



1. The airline scheduling problem

The focus for this project was in the Aircraft scheduling.

1.2.1 Aircraft Scheduling Problem

The aircraft scheduling is usually the first stage in solving the airline scheduling problem. A completed flight schedule determines the overall productivity and profitability of the airline. The design of a flight schedule involves the construction of timetables of aircraft routes (schedule planning) and the scheduling of aircraft to these timetabled routes (aircraft assignment).

Schedule planning involves determining which routes are to be flown, their frequency and the associated departure and arrival times.

A *sector* is a non-stop flight from origin to destination. A *Flight Schedule* is a list of those sectors for a fixed period of time. The combination of flight sectors to which an individual aircraft has been assigned for a certain period of the schedule is called a *Line-of-Work*. Air New Zealand creates these lines-of-work for a period of a week (Monday to Sunday)

Once the schedule is determined, the next step is to decide how to efficiently allocate resources to fly the schedule. The aircraft assignment problem involves the allocation of the particular aircraft both type (for example B737-300 or B767-200) and tail number (specific aircraft)

There are several factors that need to be looked at when making the decision about aircraft assignment. If a small aircraft flies a sector with a huge demand the opportunity to carry more passengers has been lost along with the associated revenue that could have potentially been generated. Conversely if a large aircraft was assigned to fly a sector with very little demand, fuel is wasted and high fixed landing costs are incurred unnecessarily. Also the aircraft might have been more profitably used on another route. The airline is therefore interested in an airline schedule that results in a well 'filled' aeroplane and good utilisation of existing transportation capacities. Accomplishing this match of capacity and demand for optimal revenue is the goal of the fleet assignment.

1.3 Problem Definition

In 1999 a basic optimisation package was developed for solving the international flight scheduling problem at Air New Zealand. The overall aim of this project was to look at the effect and implementation of departure time windows. This is where a flight can depart with a certain time period and the actual departure time is no longer fixed. An extra extension was to place a priority on individual sectors. This then allows sectors to be left out of the solution.

There are two ways in which the prioritising of sectors could be included. A set of flights could be provided that is unable to be covered by the current fleet. This would be the case if an airline wanted to expand into areas not already flown to. This optimisation package would return the sectors that were unable to be covered. The other approach would be to try and reduce the current fleet while still attempting to cover as much of the current schedule as possible. This is known as the fleet minimisation problem.

The outcome of this work is the construction of a flight schedule that details the sequence of flights each aircraft must perform over a period of time.

2 Model Formulation

Many of the scheduling problems that arise from the transportation industry can be posed as massive set partitioning zero-one integer programs. Due to the complex nature of the problem it is generally unrealistic to solve the model in this form using conventional integer linear programming.

To formulate and solve the international flight scheduling problem a model is required which reflects the real-world characteristics of the problem. In order to achieve the objective of constructing high quality legal and feasible schedules in a reasonable amount of time, the flight scheduling problem must be solved using mathematical optimisation methods.

2.1 Objective Functions

In general the objective function for scheduling problems is to maximise profits. In its simplest form profit is the difference between revenue and expenses.

For airlines, the physical aircraft contribute the most to both the revenue and cost components. In this way there are trade-offs that have to be made. The main source of revenue for an airline is from the sale of seats or the transportation of freight on scheduled flights. Owning, flying and maintaining aircraft on the other hand are the largest costs incurred. Also for every flight flown there is associated labour and operating costs.

The number of aircraft an airline operates is the overall driving factor in determining the number of staff required to maintain the aircraft (engineering), to fly the aircraft and tend to the passengers (technical and cabin crew) and several other essential services. Therefore the ultimate goal of this research is to minimise the total number of aircraft required to operate the international flight schedule.

2.2 Set Partitioning Model

The scheduling problem can be modelled mathematically using a generalised version of the set partitioning model. The variables can be partitioned to correspond to feasible lines-of-work for each of the individual aircraft [3]. The scheduling set partitioning problem (SPP) can be written as :

(SPP) Minimise $\underline{z} = \underline{c}^T\underline{x}$

Subject to $\mathbf{A}\underline{x} = \underline{b}$

$x_j \in \{0,1\} \qquad j = 1, \ldots\ldots, p$

where

- the elements of **A** can be defined as:

  $a_{ij} = 1$ if sector *i* is performed in line-of-work j; or

  $a_{ij} = 0$ otherwise.

- $\underline{b}$ is the right-hand-side vector and is given by:

  $b_i = 1$ for $i = 1,\ldots p.$

- The elements of $\underline{x}$ can be defined as:

  $x_i = 1$ if the line-of-work *i* is included in the solution; or

  $x_i = 0$ if the line-of-work *i* is not included in the final solution

In the context of international flight scheduling, the rows of the **A** matrix correspond to the flight sectors which must be performed by one aircraft of a particular type exactly once in the scheduled period. The columns of the **A** matrix represent feasible lines of work for a one-week period, consisting of a subset of flight sectors. A solution or schedule is given as a partition of the flight sectors by the lines-of-work.

The explicit constraint incorporated in the right-hand-side vector, $\underline{b}$, ensures that each flight sector in the designated week appears in exactly one row of the schedule solution.

The solution vector contains the solution and refers to a particular column in **A**. The optimal solution therefore will include lines-of-work that contain all sectors for the aircraft to fly with the lowest overall cost.

2.3 Expansion of the model

With the number of aircraft being used in the model unable to cover all of the supplied sectors, (either due to expansion of the number of sectors or a reduction in the fleet size) some of the sectors are not going to be able to be covered. To take this into account true slacks were added to the end of the **A** matrix. The costs associated with these lines-of-work (that contained only one sector) are related to the priority that is placed on that individual sectors. A sector with a high priority would have a high cost associated with not being covered in a line of work. The cost would be expected to be inversely proportional to the expected profit that could be generated on the sector. This eliminated any problems with infeasibility. The optimisation stage was were the fleet minimisation took place. An extra constraint was added onto the bottom of the **A** matrix to limit the number of aircraft that could be used. Each of the lines-of-work contained a one in this constraint while the true slacks contained a zero. This additional constraint has been added in figure 2.

$$\begin{pmatrix} 0\,1\,1\,1\,0\,0\,0 & 1\,0\,0\,0\,0 \\ 1\,1\,0\,0\,1\,1\,0 & 0\,1\,0\,0\,0 \\ 1\,0\,0\,1\,1\,0\,1 & 0\,0\,1\,0\,0 \\ 1\,1\,1\,0\,0\,1\,0 & 0\,0\,0\,1\,0 \\ 1\,1\,1\,1\,1\,1\,1 & \underbrace{0\,0\,0\,0\,1}_{\text{True Slacks}} \end{pmatrix} \leftarrow \text{Additional Constraint}$$

↑ Feasible lines of work

2. Matrix showing additional constraint for fleet minimisation

The user inputs the number of aircraft utilised so they can choose whether the number should be reduced.

3 Solution Method

The approach used to obtain the solution to the international flight scheduling problem was divided into three main stages: generation, optimisation and reporting of results.

3.1 Method of Next Availables

The generation stage was divided into two sub-problems. The first sub-problem of the generation process involved the method of next availables. The list of next available sectors for a given aircraft type was constructed using a complete enumeration procedures. It was designed to generate all the possible sectors that an aircraft could fly given its current port and time

Several extensions had to be added to this stage in the generation process. Each sector had to be given open and close time windows that were feasible. These time windows could vary from zero up to a maximum of 24 hours. Air New Zealand provided these time windows.

The sectors were then sorted according to the closing time of the time window. If the time window has not closed then there is the possibility that that sector could become an available. It is irrelevant at this stage what time the window opened.

Using the original data the duration for each of the sectors were calculated. This duration was then used to generate the next availables for each of the sectors. Taking each sector individually the earliest possible finish time was computed. Other sectors with start windows that were still open are then considered as possible next availables providing that they are at the same port.

For the purposes of this project the priority on each of the sectors was based on the length of the time window. This in reality would not be the case. Priority would be based on expected generated revenue, operating costs and customer demand.

Upon finishing complete enumeration, the list of next availables became the necessary input for the second stage of the generation process: the construction of the lines-of-work.

3.2 Construction of Lines of Work

The second stage of the generation process involved the construction of complete lines of work. This phase used the solutions obtained from the method of next availables. Similar to the first stage, all possible lines-of-work were constructed using a complete enumeration procedure.

Lines-of-work began with sectors that had starting times less than a specified time on Monday. The next sector that was added to the line-of-work was dependent on the current sector, and also on the *depth of subsequence* (that is, whether it was the first available sector or a sector with a greater linktime). Another factor taken into consideration was the *maximum permitted linktime* (that is, the maximum permitted time between subsequent flights). This procedure stepped forward until one of the following things occurred:

- the end of the scheduling period was attained,
- the maximum number of flights allowed in a sequence was reached,
- the sector ended after a specified time on Sunday, or
- there were no more elements left in the list of next possible sectors to add to the sequence.

When this stage was in which achieved the corresponding line-of-work was complete and this information was stored. The procedure then stepped back, removing the current sector from the sequence and adding the next sector in the previous list on next possible sectors. This process continued until all the sectors had been exhausted, without violating the user-specified parameters.

Once again there were several extensions that had to be added to this stage to deal with the incorporation of time windows. Initially the most important aspect to be added was the dynamic generation of start times for each of the sectors. This was calculated as being the earliest possible time that the aircraft became available or when the time window for the sector being added was opened. Once the fixed start time was confirmed, then finish time was calculated using the duration generated in the method of next availables.

When searching the next available file for the sector to put up into the line of work it became necessary to check that the available was still feasible (check the time window still open).

3.3 Optimisation Solution Procedure

The model developed for the international flight scheduling problem was solved using the *Revised Simplex Method*. The revised simplex method starts with and *Initial Feasible Solution* of lines-of-work that satisfy the explicit constraints of the problem. The initial solution is feasible since it satisfies the constraints implicitly, as the right hand side is by definition equal to one. However in this case it is infeasible in a practical sense because it contains only slack variables in the solution which represents an under-covering of all of the sectors.

The Revised Simplex Method moves from one current solution to an improved solution by considering all the lines-of-work that are not in the solution and deciding which would improve the solution. This is continued until no more improvements can be made to the objective function.

A fractional solution has no practical applications so a branch and bound technique is implemented to remove non-integer values from the solution. These factions were removed using branch and bound. The best integer solution found during this process becomes the optimal integer solution to the problem.

ZIP, a zero-one integer program package was used to solve this set partitioning problem. It controls all of the basic steps of the optimisation solution process for the problem under consideration.

The optimiser returns the feasible and optimal lines-of-work.

3.4 Reporting of Results

At the end of the optimisation phase, the solution was output to a file and a report-writer program written in 'C', created the international flight schedule (for each aircraft type). Essentially, this program reported the allocation of lines-of-work to aircraft (of a particular type). These results were written to a file in a format that could be read and understood by Air New Zealand software and users. This file is used in the production of the graphical solutions in the form of spider and block charts. [1]

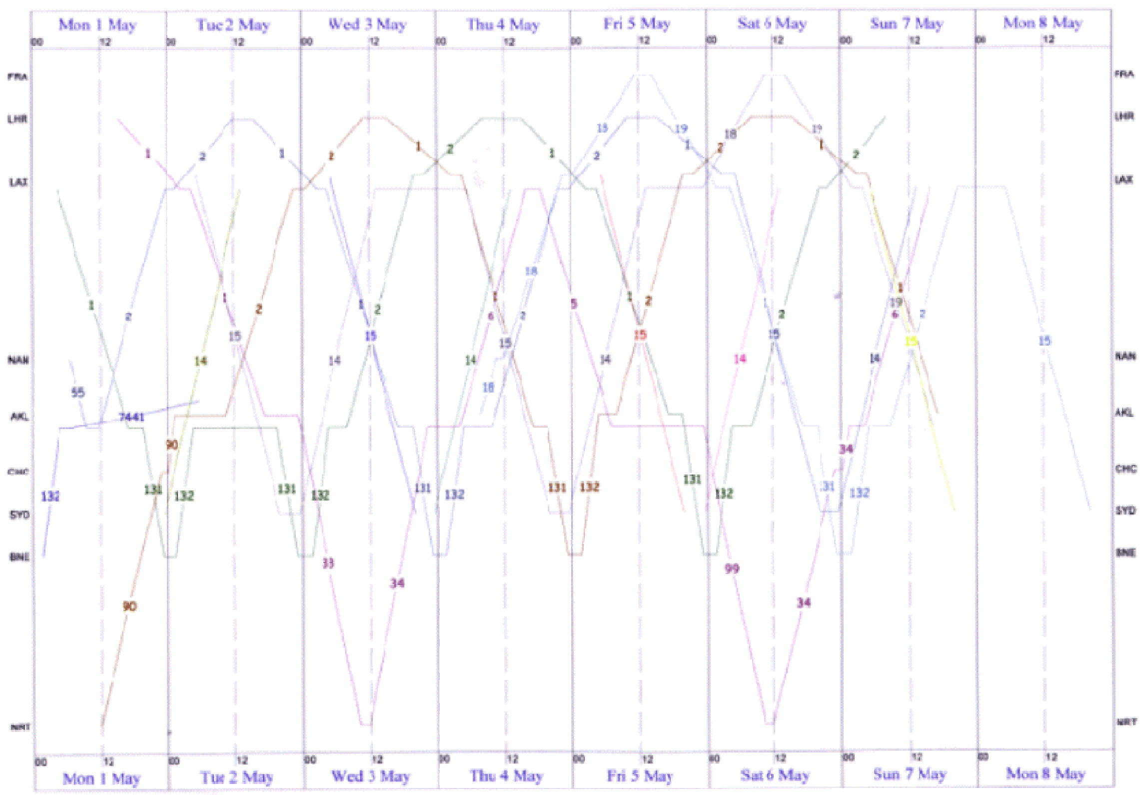
Because the in this project the start times were dynamically created the start times for the sectors in the solution had to be recalculated. This was performed in exactly the same manner as in the generation of the lines-of-work. This then added another dimension into the reporting of results because the time differences had to be taken into account to convert the arrival and departure times for each sector to local times. It was these times that had to be used in the production of results into Air New Zealand format.

4 Results and Discussion

4.1 Graphical Solutions

Solutions could only be obtained for one aircraft type using the techniques pursued in this project. These solutions were produced in two different graphical representations: spider charts and block (gant) charts. Air New Zealand, using the data contained in the output file produced by the report-writer program created both the graphical solutions.

Spider charts, illustrated in figure 3, display aircraft destinations (ports) on the vertical axis and time along the horizontal axis. Each of the ports is indicated by their associated three letter code while time is considered in terms of Greenwich Mean Time. Each sequence of connected line segments (each highlighted in a different colour) represents the path of a particular aircraft as it follows a scheduled aircraft route over the period of a week. The horizontal line segments denote periods of time when the aircraft is on the ground at an airport, and the diagonal line segments show periods of time when the aircraft is flying between airports. Every single line segment is labelled as a separate sector, with the number supplied referring to the flight number of that sector. Sectors that were omitted from the solution can be seen as line segments that are not connected to any others.

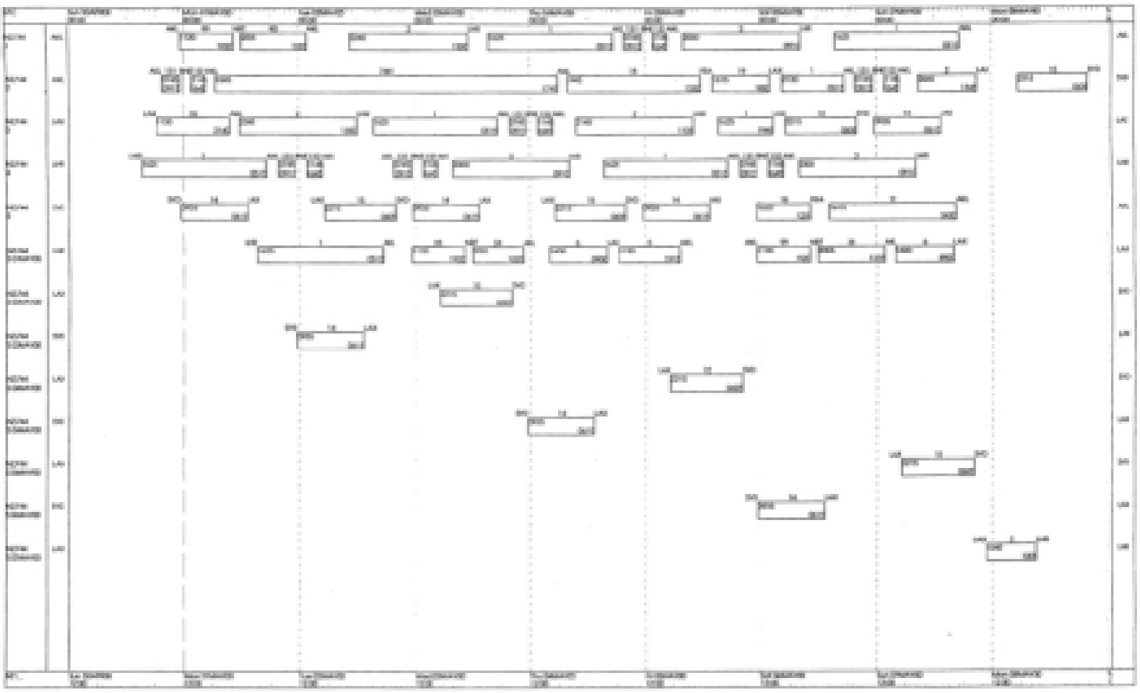


3. Spider Chart solution of Boeing 747-400 fleet

The second form of graphical representation, block charts, are much easier to follow. The optimised solution for the Boeing 747-400 fleet is shown in figure 4.

Block charts represent each sector as a single rectangular block along with the associated start and finish ports, local port start and finish times and the flight number. Similar to the spider charts each sequence of blocks indicates the path of a particular aircraft as it follows a scheduled aircraft route over the period of a week. Therefore the number of complete lines on the vertical axis represent the total number of aircraft required to operate the flight schedule for a particular aircraft type. The single blocks are the sectors that were left uncovered when dealing with the fleet minimisation problem. There are two horizontal axes, each reflecting different time zones. The horizontal axis running across the top of the page refers to the time in terms of Greenwich Mean Time while the time axis running across the bottom is in terms of New Zealand time.

The first line in the block chart is represented by the brown line is the spider chart. It begins with flight 90 from Auckland to Tokyo(NRT).



4. Block Chart of Boeing 747-400 Solution

4.2 Optimisation Results

With the inclusion of time windows the aircraft scheduling problem increased dramatically in size. This unfortunately led to the generation of 6 million variables in the **A** matrix for each of three different aircraft types. This was then too large to be solved using the optimisation techniques being implemented. The Boeing 747-400 was the only fleet where a feasible solution could be obtained. This is because this fleet contains only seven aircraft and generally performed longer sectors (long-haul) to ports on the other side of the world. Therefore the number of sectors flown in a week long period was significantly reduced.

Before looking at the fleet minimisation problem the schedule was solved just incorporating the time windows. This solution generated several lines-of-work as those contained in the fixed schedule. However when the number of aircraft was reduced better lines-of-work were chosen.

When the original solution was generated for this aircraft type the maintenance sector was one of the flights that was not covered in a line of work. This is clearly impractical, as the need for maintenance is paramount in the construction of the schedule. This was due to a low priority being placed on this sector. It did not appear to be attractive to the optimiser. The cost of not covering the maintenance far outweighed the cost of inclusion. This was then amended by increasing the cost of not performing maintenance far above the cost of any possible line-of-work.

Several attempts were made to generate a solution for the other aircraft types. Because the B767-300 and B737-300 aircraft tended to operate sectors throughout the day and have a rest period from midnight to early the next morning there were no scheduled flights between these times. Therefore the schedule could be separated into seven one-day schedules rather than trying to obtain a solution for the entire week. This approach showed real possibilities, unfortunately due to time constraints this method was unable to be implemented fully and no solutions were obtained.

5 Conclusions

Previous work has concluded that it is possible to construct a model using optimisation-based techniques to solve the international flight scheduling problem at Air New Zealand. The procedure developed in this project was successful in providing a method for generating a feasible schedule with time windows for the Boeing 747-400 fleet at Air New Zealand. It can also be said that a schedule can be created still completing significant amounts of the work, utilising one less aircraft. These techniques were also tried with the other aircraft types that are currently owned by Air New Zealand, but unfortunately the problem sized increased significantly to the point where solutions were unable to be obtained. The results obtained from optimisation runs performed using actual data from Air New Zealand have proved satisfactory for the 747 fleet.

In order for results to be obtained in the future a different method of attack needs to be implemented. Dynamic column generation is one of the techniques the might proved feasible in the future. This would then reduce the problem size without effecting feasibility and optimality yet still be able to produce quality solutions.

Intimate knowledge of and experience with the airline industry are two implicit components of the manual design process of the international flight schedule at Air New Zealand. In order for these computer optimised solutions to be competitive with the current manual solutions, significant extensions would need to be made to this model to reflect these implicit components.

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References

[1] Allbon, C. (1999) 'Optimisation of Air New Zealand's International Flight Schedule'. Year Four Project Report, Department of Engineering Science, The University of Auckland.

[2] Day, P. (1996) 'Flight Attendant Rostering for Short-Haul Airline Operations'. PhD Thesis, Department of Engineering Science, The University of Auckland.

[3] Ryan, D.M. and Meyer, J.A. (1993) 'Aspects of Crew Rostering Systems for Air New Zealand International Air-crews'. Department of Managements and Information Systems, University of Auckland, Auckland.