# Capital Planning in the Paper Industry using COMPASS

Graeme Everett Sami Aoude Andy Philpott
Norske Skög and
Department of Engineering Science
University of Auckland
graeme.everett@norske-skog.co.nz

## Abstract

Norske Skög operates three paper mills in Australia and New Zealand: two paper machines at the Boyer mill in Tasmania, one machine at the Albury mill in Victoria and three machines at the Tasman mill in the Bay of Plenty. Decisions on upgrading the paper machines or replacing them must take account of changing demand for different grades of paper. In upgrading the machines there are many different choices of technology, each of which provides different production possibilities and has different costs. This paper describes the development and implementation of a model used by Norske Skög to assist in determining an optimal capital plan. The model, called COMPASS, is a large mixed integer programming problem formulated with a ten-year planning horizon, and solved using AMPL and Cplex 7.1. We present some of the special features of this model and describe how it was used in the capital planning process.

## 1 Introduction

In a recent paper [2], Everett and coauthors discuss a model for capacity planning under uncertainty in the paper industry. This model, known as SOCRATES, is a large mixed integer programming model. SOCRATES was designed to help Fletcher Challenge Canada construct capital expansion and improvement plans for each of their six paper machines on Vancouver Island.

COMPASS is a similar model to SOCRATES in many respects. There are a number of areas of difference betwen the two models, and in this paper we will focus on two of them:

1. Pulp recipes are allowed to vary depending on capital investment decisions.
2. The objective function for COMPASS is to maximize after tax cash flow.

The paper is laid out as follows. In the next section we give a basic model description. In section 3 we outline how COMPASS treats pulp recipes. Section 4 discusses market demand for different grades of paper, and section 5 discusses how the COMPASS model deals with different tax rates in Australia and New Zealand. In section 6 we discuss project management issues and the use of model results.

# 2 Basic Model Description

In this section we outline the basic structure of the COMPASS model. Since this is essentially identical to the SOCRATES model [2], we provide only a brief description. Suppose there are $N$ mills indexed by $n$, $M$ paper machines indexed by $m$, $C$ capital items indexed by $c$, $J$ paper grades indexed by $j$ and $T$ planning years indexed by $t$. Apart from these index definitions, we adopt the convention throughout this paper of using lower case Roman letters to denote parameters, upper case Roman letters to denote continuous variables, and lower case Greek letters to denote binary variables. The central decision variables in this model are the nonnegative variables:

$$X_{mjt} = \text{tonnes of paper of grade } j \text{ produced on machine } m \text{ in year } t,$$

and the binary decision variables

$$\sigma_{mct} = \begin{cases} 1, & \text{if capital item } c \text{ is installed on machine } m \text{ in year } t, \\ 0, & \text{otherwise.} \end{cases}$$

We proceed to show how the different aspects of COMPASS are formulated using these and other variables.

## 2.1 Capital Requirements and Costs

It is convenient here to define the set $S$ indexed by $s$ being equal to the set of planning years $T$ plus one additional year. We assume that capital item $c$ is installed at most once, so

$$\sum_{s \in S} \sigma_{mcs} = 1, \quad m = 1, \ldots, M, \quad c = 1, \ldots, C.$$

This means that for each $t = 1, 2, ..., T$, the expression

$$\sum_{s \leq t} \sigma_{mcs} = \begin{cases} 1, & \text{if capital item } c \text{ is available on machine } m \text{ in year } t, \\ 0, & \text{otherwise.} \end{cases}$$

(Observe that as $S$ includes one more year than $T$, so a machine that is not closed will in fact set $\sigma_{mcs} = 1$ in the year $s = T + 1$.)

Similarly machines can only be shut down at most once, so for each $s = 1, 2, \ldots, T + 1$, we define binary decision variables

$$\alpha_{ms} = \begin{cases} 1, & \text{if machine } m \text{ is shut down in year } s, \\ 0, & \text{otherwise.} \end{cases}$$

and require

$$\sum_{s \in S} \alpha_{ms} = 1, \quad m = 1, \ldots, M.$$

Therefore for each $t = 1, 2, ..., T$, the expression

$$\sum_{s > t} \alpha_{ms} = \begin{cases} 1, & \text{if machine } m \text{ is available in year } t, \\ 0, & \text{otherwise.} \end{cases}$$

In addition to the decisions determining capital and machine closure we require binary decision variables determining which grades are produced on given machines. Let

$$\rho_{mjt} = \begin{cases} 1, & \text{if grade } j \text{ is produced on machine } m \text{ in year } t, \\ 0, & \text{otherwise.} \end{cases}$$

Then we can only produce on machines that have not been closed,

$$\rho_{mjt} \leq \sum_{s > t} \alpha_{ms}, \qquad m = 1, \ldots, M, \qquad t = 1, \ldots, T,$$

and on machines having the required capital;

$$\rho_{mjt} \leq \sum_{s \leq t} \sigma_{mcs}, \tag{1}$$

if machine $m$ requires capital item $c$ to produce grade $j$. The cost of capital item $c$ for machine $m$ is denoted by $g_{mc}$. Since this is incurred on average a year before deployment, we add the term $\sum_{m \in M} \sum_{c \in C} g_{mc} \sigma_{mct+1}$ to the investment cost incurred in year $t$.

## 2.2 Paper Machine Expansion Decisions

Paper production is constrained by market demand (as described in [2]) and paper machine capacity limits. It is possible to upgrade each paper machine by replacing sections forming bottlenecks thereby increasing the machine capacity for all products. We define binary decision variables

$$\delta_{mt} = \begin{cases} 1, & \text{if machine } m \text{ is upgraded in year } t, \\ 0, & \text{otherwise,} \end{cases}$$

and require that

$$\sum_{s \in S} \delta_{ms} = 1, \qquad m = 1, \ldots, M.$$

We define parameters $u_m$ being the percentage of additional capacity that is available from each upgrade project, for which there is an associated capital cost denoted by $h_m$. Each machine will have an annual capacity for each grade of $a_{mjt}$ tonnes plus an additional percentage defined by $u_m$ if the decision has been taken to upgrade the machine in the present, or earlier year. This gives the following inequality:

$$\frac{X_{mjt}}{a_{mjt}} \leq 1 + \sum_{s \leq t} \delta_{ms} u_m, \quad m = 1, \ldots, M, \qquad t = 1, \ldots, T. \tag{2}$$

We restrict production of grade $j$ in year $t$ to those machines $m$ that have invested in capital items necessary to make grade $j$, thereby allowing $\rho_{mjt} = 1$ by equation (1). This is modelled by

$$X_{mjt} \leq \rho_{mjt} \bar{a}_{mjt} \qquad m = 1, ..., M, \quad j = 1, ..., J, \quad t = 1, ..., T.$$

where $\bar{a}_{mjt}$ is an appropriate upper bound for machine capacity. (In fact equation (2) was developed further to reflect the fact that some capital options as defined by $\sigma_{mcs}$ had implications for the capacities of other products, but in the interests of brevity we will not discuss this here.)

# 3 Recipes

Each of the paper machines uses a particular recipe of differing pulp types and chemical additives (referred to as furnish) for each paper product it is capable of producing. An important consideration in this project is the possibility of switching to a different furnish that is significantly cheaper if certain capital items are improved, added or replaced. The present configuration of the pulp plants at each of the three mills has been outlined in [3], and is summarised in Table 1, which shows the types of pulp that are available at each mill.

| | CCS | RCF | SGW | TMP | RMP |
|---|---|---|---|---|---|
| Albury | | √ | | √ | |
| Boyer | √ | √ | | √ | |
| Tasman | | | √ | √ | √ |

Table 1: Pulp plant capabilities at each mill

Other components of the furnish supplied to the paper machines are kraft pulp, starch, clay fillers, carbonate fillers, chemicals and water. These furnish components comprise the set $F$, indexed by $f$. We define binary variables

$$\mu_{nfs} = \begin{cases} 1, & \text{if furnish } f \text{ is available at mill } n \text{ in year } s, \\ 0, & \text{otherwise.} \end{cases}$$

Each furnish component incurs a fixed cost that can be avoided if $\mu_{nfs} = 0$. We define the parameter

$$k_{nf} = \begin{cases} \text{capacity of pulp plant } n, & \text{if pulp furnish } f \text{ is available at mill } n, \\ 0, & \text{otherwise.} \end{cases}$$

Then superflous binary variables can be removed by the inequalities

$$\mu_{nfs} \leq k_{nf}, \quad n = 1, \ldots, N, \qquad f = 1, \ldots, F, \quad s = 1, \ldots, T+1.$$

Decisions to build additional pulping capacity are considered for a subset of $F$. We let $E$, indexed by $e$, be this subset of $F$, and define binary variables

$$\nu_{nes} = \begin{cases} 1, & \text{if pulp plant } e \text{ is expanded at mill } n \text{ in year } s, \\ 0, & \text{otherwise.} \end{cases}$$

We also require

$$\sum_{s \in S} \nu_{nes} = 1, \quad e = 1, \ldots, E, \quad n = 1, \ldots, N.$$

Therefore for each $t = 1, 2, \ldots, T$, the expression

$$\sum_{s \leq t} \nu_{nes} = \begin{cases} 1, & \text{if the new pulp plant associated with furnish } e \\ & \text{is commissioned in year } t, \\ 0, & \text{otherwise.} \end{cases}$$

We denote the cost of the expansion of the pulp capacity by $q_{ne}$, which results in a contribution of $\sum_{n \in N} \sum_{e \in E} q_{ne} \nu_{net+1}$ to the total investment cost in year $t$.

Recipes are defined for each product at each paper machine by the parameter $b_{mjf}$ with the condition that the sum of furnish percentages must equal 100%:

$$\sum_{f \in F} b_{mjf} = 1, \; m = 1, \ldots, M, \; j = 1, \ldots, J.$$

The amount of each furnish component required is determined by the production plan. We define furnish usage variables $Y_{mft}$, being the tonnes of furnish $f$ used on machine $m$ in year $t$.

$$Y_{mft} = \sum_{j \in J} b_{mjf} X_{mjt}, \quad m = 1, \ldots, M, \quad f = 1, \ldots, F, \quad t = 1, \ldots, T. \tag{3}$$

However it is possible to alter recipes if certain capital options are selected. For instance at the Tasman mill it would be possible to replace expensive kraft pulp with cheaper TMP pulp if an additional line of TMP was installed coupled with closure of the SGW mill. We now have a new set of recipes $R$, indexed by $r$, and expanded parameter and variable definitions so that equation 3 becomes:

$$Y_{mft} = \sum_{r \in R} \sum_{j \in J} \hat{b}_{mjfr} \hat{X}_{mjrt}, \quad m = 1, \ldots, M, \quad f = 1, \ldots, F, \quad t = 1, \ldots, T.$$

To ensure that furnish satisfies its capacity constraints we require

$$\sum_{m \in M[n]} Y_{mft} \leq k_{nf}, \quad n = 1, \ldots, N, \quad f = 1, \ldots, F, \quad t = 1, \ldots, T,$$

where $M[n]$ is defined to be the set of machines at mill $n$.

Paper production is now defined as the sum of production made according to each recipe:

$$X_{mjt} = \sum_{r \in R} \hat{X}_{mjrt}.$$

The conditions under which each recipe can, or must, be used are many and complex, and will not be outlined here. In the model constraints on the $\hat{X}_{mjrt}$ variables were constructed according to the values of combinations of binary variables.

### 3.1 Raw Material Consumption

During the process of making pulp and converting it into paper on the paper machines a small percentage of fibre is lost. This fibre loss varies by mill and fibre type according to the parameter $l_{nf}$. Raw materials such as wood are required to produce each type of pulp and each pulping process has a yield $y_{nf}$. In the case of fillers and chemicals which are pure additives the yield is equal to 1. The amounts of raw materials required are defined by the variables $R_{mft}$ as follows

$$R_{mft} = Y_{mft}(1 + l_{nf})y_{nf}.$$

Each raw material is available at a cost $w_{mf}$ and the total cost of raw materials incurred each year is given by $\sum_{m \in M} \sum_{f \in F} w_{mf} R_{mft}$.

## 4 Market Demand and Manufacturing Costs

Estimates of market demand and prices for the various products within Norske Skög's manufacturing capability were provided by the company's marketing department. COMPASS includes constraints to ensure that deliveries can not exceed market demand, and that the appropriate sales revenue is gathered. Paper machines, mills and pulp plants that are not permanently closed incur fixed costs. Additional fixed costs were incurred as a result of some capital projects.

Both the paper machines and the pulping plants incur variable costs. Paper machine variable costs in some cases were altered by capital decisions. New pulping plants operate at an increased efficiency, and at a lower cost than the existing plants.

The paper making variable costs differ depending on the recipe used, as defined by the parameters $z_{mjr}$. Variable paper making costs, $V_{mt}$, are calculated by summing the tonnes made by each recipe for each product.

$$V_{mt} = \sum_{j \in J} \sum_{r \in R} z_{mjr} \hat{X}_{mjrt}.$$

## 5 Tax

COMPASS maximizes discounted cash flow after company tax has been deducted. New Zealand and Australia have different company tax rates. This means that the country in which taxable revenue is earned is an important factor in determining capital expenditure. We thus associate with each country its own earnings stream that is taxed and discounted before being added together to give an overall discounted cash flow.

To describe how tax is modelled for each country in COMPASS, we define for each country and each year $t$ the following non-negative variables (the country index is suppressed for notational convenience).

$$\begin{array}{rcl} A_t &=& \text{earnings after tax and capital spend} \\ B_t &=& \text{earnings before tax and depreciation} \\ E_t &=& \text{earnings before tax} \\ r_t &=& \text{tax rate} \\ d_t &=& \text{depreciation rate} \\ I_t &=& \text{capital investment} \\ K_t &=& \text{book value of capital stock at the end of } t \\ W_t &=& \text{taxable earnings} \\ S_t &=& \text{capital written off} \\ L_t &=& \text{loss incurred} \end{array}$$

We first add up the total investment $I_t$ in capital in each year $t$ to obtain

$$I_t = \sum_{m \in M} \sum_{c \in C} (g_{mc}\sigma_{mct+1} + h_m \delta_{mt+1}) + \sum_{n \in N} \sum_{e \in E} q_{ne}\nu_{net+1}.$$

(Here we assume throughout that the investment is incurred one year before the new plant is commissioned.) The book value of the capital stock will obey the following dynamics:

$$K_t = (1 - d_t)K_{t-1} + I_t - S_t.$$

Following [2] the earnings $B_t$ before tax and depreciation is the sum over all products and markets of the revenue earned minus production variable costs, transportation costs, raw material costs, and annual fixed costs. Subtracting depreciation yields

$$E_t = B_t - d_t K_{t-1}.$$

To compute the taxable earnings $W_t$, we now let $Z_t$ be an unrestricted variable which measures the company's net profit in period $t$, which is the difference between their current earnings $E_t$ and losses $L_t$, adjusted for any losses brought forward from previous years. If $L_0$ is an initial tax loss brought forward from previous years then we have

$$Z_1 = E_1 - L_1 - L_0,$$

and for $t > 1$, the losses brought forward from previous years are

$$\sum_{s \leq t-1} (E_s - L_s - W_s),$$

giving

$$Z_t = \sum_{s \leq t} (E_s - L_s) - \sum_{s \leq t-1} W_s.$$

At any period $t$ the taxable income will be zero if we have accumulated losses, so

$$W_t = \max\{Z_t, 0\} \tag{4}$$

which gives the net earnings after tax as

$$A_t = E_t - r_t W_t - I_t.$$

The objective function of COMPASS is now the present value of the cash flow stream $A_t$, $t = 1, \ldots, T$ plus a terminal value summed for each country. With a strictly positive discount rate, early payment of tax is discouraged, so the condition (4) can be modelled by the linear inequality

$$W_t \geq Z_t,$$

since the optimization will attempt to keep $W_t$ as small as possible. (We observe that in the absence of discounting, this approach can lead to erroneous alternative solutions being reported, when for example $W_1$ is made very large and the excess tax payment is carried forward as a tax loss.)

# 6 Results

## 6.1 Project Management

To ensure the success of the project the following elements had to be satisfied:

1. correct specification of the project frame of reference, i.e. precise definition of the problem to be solved;
2. proper skilling of the team;
3. ownership of the input and results by the team;
4. credibility of the inputs, methodology and therefore the results.

The senior management team in the region owned the project and specified the original frame. A steering committee comprising members from the senior management team provided input to the project. The steering committee was empowered to resolve problems as they arose and to alter the frame as necessary. The steering committee received regular feedback on progress. Presentations were made to the steering committee as results became available, and the team addressed any comments and criticisms arising from these presentations. This helped gain credibility for the project by ensuring that relevant issues were addressed and perceived shortcomings corrected throughout the project, thus avoiding surprises at the end.

The project team comprised a project manager, a representative from each mill with engineering and process knowledge, a marketing expert, a product development expert, and an operations research specialist. The team could draw upon other resources from the mills.

The COMPASS model was a convenient method not only of analysing the various strategic options but also for collating the large amounts of data and assumptions that were critical to the project. The model inputs were facilitated by an Excel spreadsheet making use of the ODBC feature in AMPL 7.1 [1]. This spreadsheet was located on a shared server so that all project members could see all of the input data. Scrutiny of other mill's production data was encouraged and in fact a number of combined working sessions were held. This contributed to the team owning the input and eventually the results.

The regular face-to-face meetings and conference calls held throughout the duration of the project helped ensure that the team worked together to achieve the project goals. It was important that the modelling expert understood all the relevant details at each mill and incorporated them accurately in the model. It was equally important that the project members understood how the COMPASS model worked and especially how to understand and interpret the output. Both of these were successfully achieved, and as a result COMPASS earnt a great deal of credibility and respect amongst the project team, the steering committee and others involved with the project.

## 6.2 Use of COMPASS

In the first instance, a number of strategic scenarios were identified for COMPASS to solve. The results of the various model runs were compared and this enabled the project team to understand the important strategic decisions the company needed to consider. Often results of model runs generated further questions and these were generally easily answered by defining a further scenario (by restricting variables) and using COMPASS to identify key differences.

Several market data scenarios were also developed and COMPASS was used to find the optimal decisions under each of these, and also the optimal decisions given a subset of the scenarios used in the base case. No attempt was made in this project to expand COMPASS into a stochastic optimization model as the deterministic version struggled to find optimal solutions, and increasing the number of binary variables would have rendered the model unsolvable.

## 6.3 Performance of COMPASS

COMPASS is a very large mixed integer programming problem with approximately 1800 binary variables, 24,000 linear variables and 20,000 constraints. Most scenarios were solved to bound gaps of less than 1% in 24 hours using a PC with a 1GHz Pentium III processor and 512MB RAM.

# 7 Conclusions

The successful use of COMPASS in Project MAPS is another example of the use of powerful optimization tools within a large corporate company. The development and use of COMPASS was a major challenge due to the many complex business rules that needed to be included. It is difficult to see how this challenge could have been met without a flexible mathematical programming system (in this case AMPL 7.1). The success of this modelling exercise can be mainly attributed to the excellent project management, and the high level of skill and knowledge of the project members.

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