A new framework for combat risk assessment using the MANA model

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Abstract

Many militaries use combat modelling as an Operational Research technique to assess potential risks for various kinds of operations. However, conventional combat equations tend to be firepower centric and of limited use for describing the environment that the modern warfighter must operate in. Significantly, they fail to explore how armed forces can adapt and re-organise themselves on the battlefield to defeat an apparently superior opponent, but rather treat each side as little more than "cannon fodder". A new equation-based method is suggested which deals with dispersed land warfare operations. A generic scenario of a small but powerful Blue force maneuvering through a larger but dispersed Red force is used to illustrate this theoretical framework. The scenario is run within an advanced cellular automaton combat model called MANA, designed by the New Zealand Defence Technology Agency (DTA). The results show that a certain ensemble of runs will never reach a pre-determined casualty level. For those that do, the role of individual lethality is not as crucial as might be expected from the Lanchester equation, due to the ability of model entities to concentrate firepower. This is symptomatic of the correlated nature of the distribution of entities, both in space and time. These correlations are describable by fractal power laws. Furthermore, the distribution of model outcomes exhibit a fat tail, as is characteristic of other systems which display fractal traits. The model results appear to support the suggested equation..

1 Introduction

This paper endeavors to introduce a new equation-based method for estimating and determining losses in land conflict that can address types of operations for which the widely used Lanchester equation (Lanchester, 1914 [6]) is inadequate.

The suggested equations are tested using a DTA-designed model called MANA. Moreover, the model is used here to illustrate a theoretical framework for describing complex and non-linear battlefields using fractal methods. This is done by exploring the spatial and temporal correlations within the population of entities, which arise as the result of multiple interactions of the personalities of the combatants.

By contrast, consider the Lanchester equation:

$$\begin{aligned} \frac{dR}{dt} &= -k_B B(t), \quad R(0) = R_0 \\ \frac{dB}{dt} &= -k_R R(t), \quad B(0) = B_0 \end{aligned} \qquad (1)$$

where R and B represent the numerical strength at time t of opposing Red and Blue forces, and k_B and k_R the killing rate of a Red/Blue individual. Equation 1 is a linear attrition model that assumes homogeneity among the combatants, and ignores the effects of spatial and temporal correlations.

The work here focuses on a generic dispersed "battlefield", where a small but powerful Blue force must maneuver its way through a much larger but dispersed and (in most cases) individually less powerful Red force.

Such a scenario may represent a range of real operations taken from events in recent years. At one end of the spectrum, it may be the situation in Mogadishu in 1993 where a small group of elite forces found themselves having to maneuver through an urban environment filled with hostile Somalis. At the other end, it may represent situations in East Timor where UN aid workers suddenly found themselves swamped by desperate locals cueing for scarce jobs. In such a case, "kill probabilities" should be viewed as "injury probabilities".

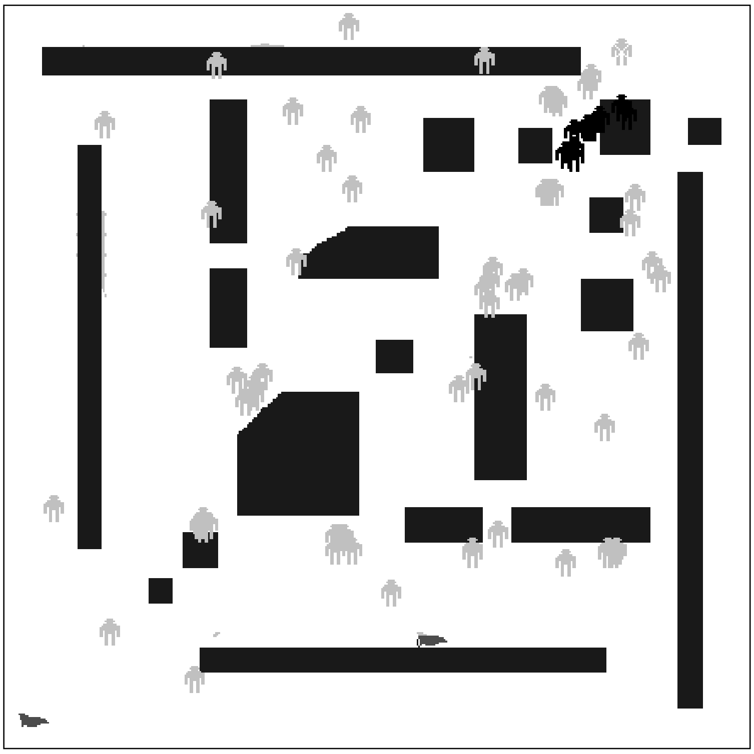

Figure 1: Screen shot of the MANA model scenario analyzed.

Figure 1 shows the appearance of the MANA model, as used in this study. The entities in the model are divided into two groups with distinct personalities:

Blue troops:

- Try to move from the top right-hand side of the screen down to the bottom left-hand side of the screen.
- Stay with friends, but maintain a certain minimum spacing if possible.
- Avoid Red.
- Have firepower equal or superior to Red (single-shot kill probability (SSKP) of 0.2).

Red hostiles:

- Move toward friends if in a group of less than 3.

- Move toward Blue when they see them.
- Red firepower is systematically varied from SSKP = 0.01 to 0.2.
- Require a local (i.e. within detection range) numerical advantage of more than 3 to attack Blue.

This last requirement represents the “intangible” concept of mana, as in the Maori word meaning an aura of authority and respect. The assumption is that a foe will not recklessly attack his enemy if he has respect for (or fear of) him. Rather, he will wait until an opportune moment (i.e. when a substantial numerical advantage exists).

The MANA model is, essentially, a cellular automaton (Ilachinski, 2000 [4], 2001 [5]; Hoffman and Horne, 1998 [2]; Hunt, 1998 [3]; Woodcock et al. 1988 [9], 1989 [10];) combat model, though with the key difference from traditional cellular automaton models that it allows global interactions.

Each automaton is governed by a set of weightings that determine its propensity to move toward/away from friends/enemies and goals or waypoints. These are modified by requirements for the automata to keep a minimum distance from these goals, or to only attack enemies when a certain numerical advantage exists, or to only advance in the company of a certain number of friends. Here, we will not go into any further detail about the model due to the amount of space that would be required to fully describe it.

2 Analysis

Cellular automata models are known to produce power-law, or “fractal”, distributions, as highlighted by the work of Bak et al (1989) [1], so that we expect the patterns into which the entities in the MANA model to evolve are likely to be able to be characterized by a fractal power law. What is meant by this is that the pattern has fine structure to it, rather than being a perfect column or other simple shape. Virtually any object that possesses fine structure will display some sort of fractal scaling range for some range of scales. However, real objects only display a limited scaling range – perfect fractals with indefinite scaling ranges only exist as mathematical abstractions.

Such patterns can be characterized by a fractal dimension, *D*, defined as:

$$D = \lim_{d \to 0} \frac{\log N}{\log\left(\frac{1}{d}\right)} \quad (2)$$

where *N* is the number of boxes of side length *d* that are required to completely cover the distribution (Mandelbrot, 1983 [8]). Note that this equation may be re-expressed as a power law:

$$N(d) \propto d^{-D} \qquad (3)$$

If for some range of *d* the value for *D* is non-integer, then the patterns may be said to behave as a fractal for that range.

Equation 2 is easily adapted to the MANA data, since we need simply count how many cells contain entities, then aggregate cells and count again, repeating this process several times. However, there are also difficulties in trying to obtain such a measure. For one, the distribution of the entities is constantly changing. Furthermore, we are mostly interested in the distribution during combat, since this is when attrition occurs. We might instead record the locations of casualties and obtain a fractal dimension of this instead. But for a single run, the number of casualties provides a relatively sparse pattern to try to analyze.

An alternative is to analyze the collected locations of multiple runs. Given a large enough set of runs, one would expect that eventually every cell would have at least one kill occur it in. Therefore, to use this method, we must use a fixed number of runs to generate patterns for each of the different cases.

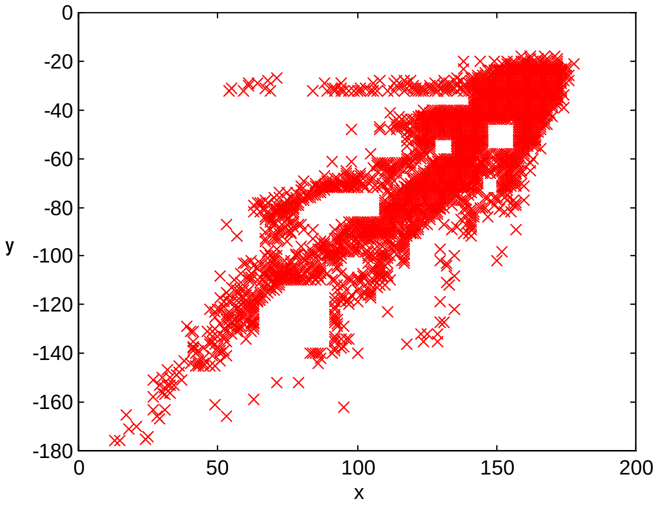

Figure 2: Cumulative distribution of casualty positions from 600 runs for case III.

Figure 2 shows all the locations of kills for 600 runs of case iii, where we have set Red’s SSKP to 0.1. Analyzing how the number of cells containing kill locations changes as the size of the cells increases produces the log-log plot in Figure 3. The slope of this plot is the fractal dimension, *D*, which in this case is 1.40.

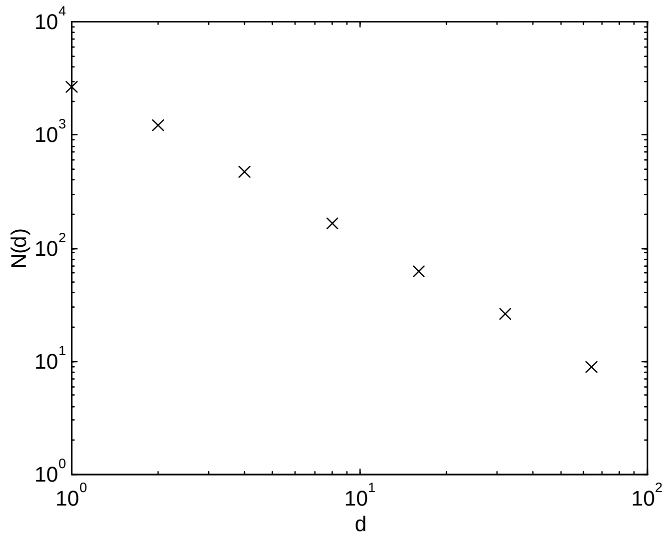

Figure 3: The dependence of number of boxes needed to cover all casualty locations on the size of the box used. The slope of the straight line here is – 1.4, which is the fractal dimension.

Given this characterization of the distribution of the forces during engagements, we can now postulate as to the attrition rate for Blue. If *B* is the number of remaining Blue, and *R* is the remaining number of Red, then the attrition rate should only depend on:

$$\frac{\Delta B}{\Delta t} = f(R,t,k,D) \qquad (4)$$

where *t* is time and *k* is the single-shot kill probability for Red shooting at Blue.

As discussed in the introduction, the Lanchester equation does not take account of spatial and temporal correlations. The spatial correlations are described by *D*. It is therefore postulated that not only does the spatial distribution display a power law dependence in *d*, but the temporal structure function also obeys a power-law:

$$\left\langle \left| B(t_0 + \Delta t) - B(t_0) \right|^2 \right\rangle \propto \Delta\ t^{F(D)} \qquad (5)$$

where we expect *F* (*D*) is a non-integer. In order for this equation to be dimensionally correct, the right-hand side needs to be multiplied by another power of a unit with dimensions of time.

From equation 4, only *k* has dimensions of time, suggesting an equation of the form:

$$\left\langle \left| B(t_0 + \Delta t) - B(t_0) \right| \right\rangle \propto R\, k^{F(D)}\ \Delta t^{F(D)} \qquad (6)$$

This form is appealing because setting *F* = 1 and dividing both sides by an increment of *t* reduces the equation back to the Lanchester equation. Note that for a given run, the Red automata may or may not evolve into a pattern with a fractal dimension similar to the majority of the other runs. Suppose we are interested in how quickly the Blue casualty level reaches some pre-decided level. For the ensemble of runs which reach this level, the distribution into which the Red automata evolve must be sufficiently dense to allow this level of casualties to be caused. Hence, by selecting just this ensemble, it seems reasonable that we are also selecting runs with similar fractal dimensions for the distribution of forces. We add the condition to equation 6 that the angled brackets represent an average over the ensemble of runs which actually reached this pre-determined level of casualties.

2.1 The *k* part

The right-hand side of equation 6 has two implications. The first relates to the *k* variable. The equation suggests that the rate of Blue attrition should depend on *k* to a non-integer power-law.

Figure 4 shows the attrition rate for Blue as a function of *k* for two cases, one without Red having communications (I) and one with Red communication (III). Each point on the plot was calculated from 600 runs of the specific variation. Note that attrition rate is calculated by finding the mean time to reach four casualties (or other arbitrary casualty level), for just those cases that reach this level. Although not as convincing a fit as seen in our earlier studies, nonetheless a power-law fit (i.e. a straight line on the log-log plot), seems to be a reasonably good approximation. Note that since reducing Red firepower requires Red to cluster to a greater extent to do the same damage to Blue, one cannot expect the fractal dimension for the Red distribution to necessarily remain the same for all values of *k*. If *D* is a function of *k*, then it would be reasonable to expect departure from a perfect straight-line power law in Figure 4.

Figure 5 plots the number of runs for which four casualties occur (out of 600). Here, there appear to be at least two distinct regions: i) the left-hand side of the graph, displaying a steep, approximately power-law, slope, and; ii) the right-hand side, which is "flat" and independent of *k*. Obviously the right-hand side represents situations where nearly all runs produce casualty levels of four or greater. As one moves to the left on the graph, the plotted points gently slide away from this level, indicating a slowly growing number of runs for which casualties are light.

At the left-hand side of the figure, the number of runs reaching four casualties falls dramatically, in this case, approximated by a power law, i.e.:

$$N(C \geq 4) \propto k^{1.88} \qquad (7)$$

However, this slope depends very much on the points chosen might vary anywhere from an exponent of 1 to 2. Nevertheless, equation 7 may make a useful first-order approximation.

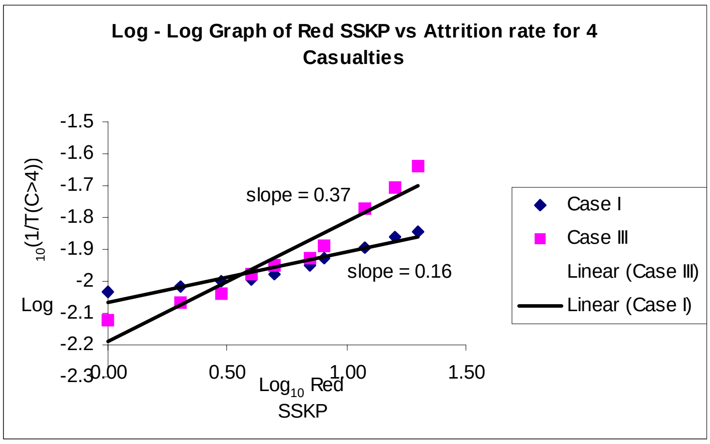

Figure 4: Attrition rate (more accurately, 1/time to reach four casualties) as a function of Red lethality.

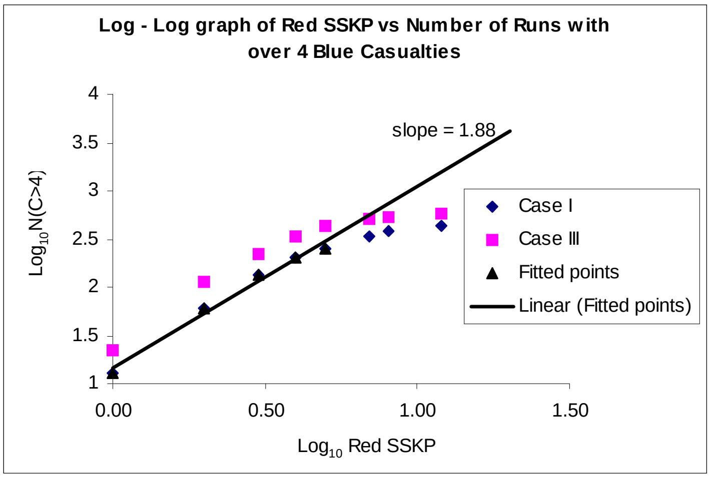

Figure 5: Number of runs (out of 600) that reach four casualties as a function of Red *k*.

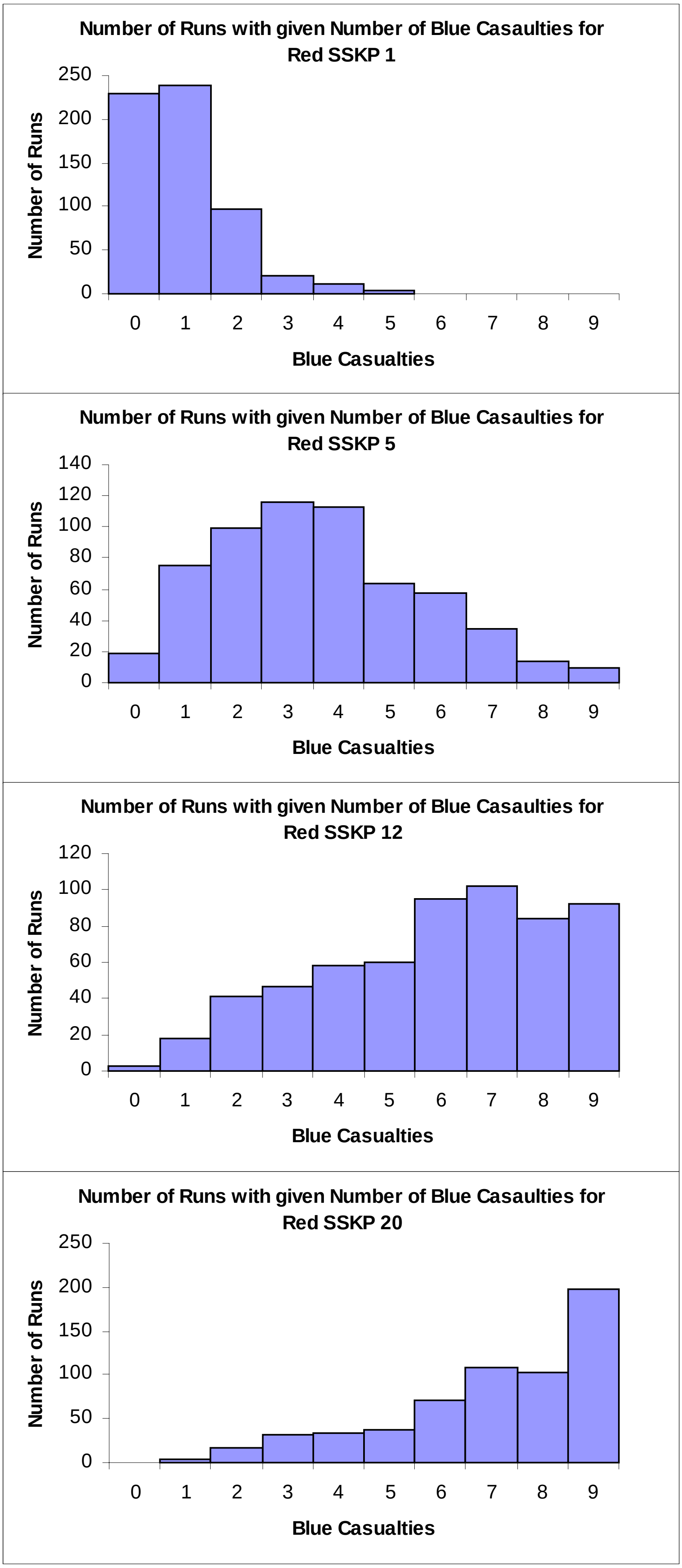

Figure 6: Distribution of casualty levels for case I, as a function of *k*.

The results presented in the figures display behavior significantly different from the Lanchester case in two important ways: i) for the ensemble of runs that do reach the four casualty level, the rate of attrition is less strongly dependent on *k* than for the Lanchester model of attrition; ii) only a certain portion of the runs reach this level … in fact, some runs may have virtually no casualties, while others have extremely heavy casualties, despite having the same *k* values.

The second point highlights the issue of so-called fat-tailed distributions of outcomes that models such as MANA display. That is, there is a disproportionate chance of extreme events when compared with a normal (Gaussian) distribution. This is illustrated in Figure 6 for several values of *k*, for case I. A good example is the case where $k = 0.2$, where there is a long tail of possible outcomes extending down to the possibility of only one or two casualties, despite the heavy majority of runs producing nearly 100% casualties.

2.2 The *t* part

The second implication of equation 6 relates to the degree of temporal correlation that exists in the casualty data (or, indeed, any data related to interaction between the forces, such as contact reports). If the temporal structure function is as equations 5 and 6 imply, then one may expect the spectrum of the casualty function to exhibit a power law of slope $\beta = -(2F + 1)$. However, it is not clear how one analyzes the function $B(t)$ here, since once again the number of Blue entities is small and does not constitute much of a signal to characterize. A simpler approach is to record the time of every casualty in all of the 600 runs, and build a graph of number of casualties from all runs at a given time step. Such a graph appears as in Figure 7, using the recorded time of 250 casualties.

The degree of temporal correlation can be characterized by taking the power spectrum of the data. Figure 8 shows the power spectrum of this casualty time series. It is notable that the power-law exists on the left hand side, while the right hand side displays a flat, white noise spectrum. This is extremely reminiscent of turbulence power laws, which typically terminate in a flat "dissipation scale" region of the spectrum. The power law is found by fitting a straight line to the left-hand side of the spectrum, as in the figure.

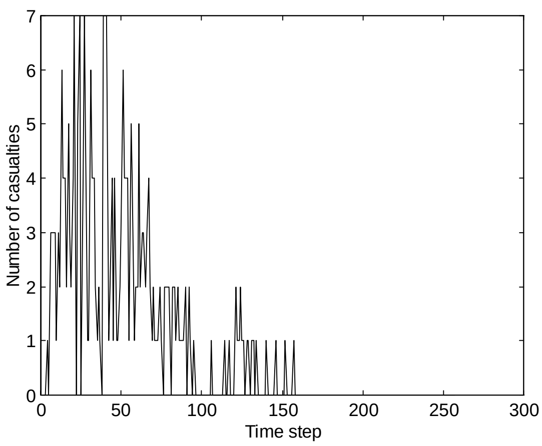

Figure 7: Temporal distribution of 250 casualties obtained from several runs.

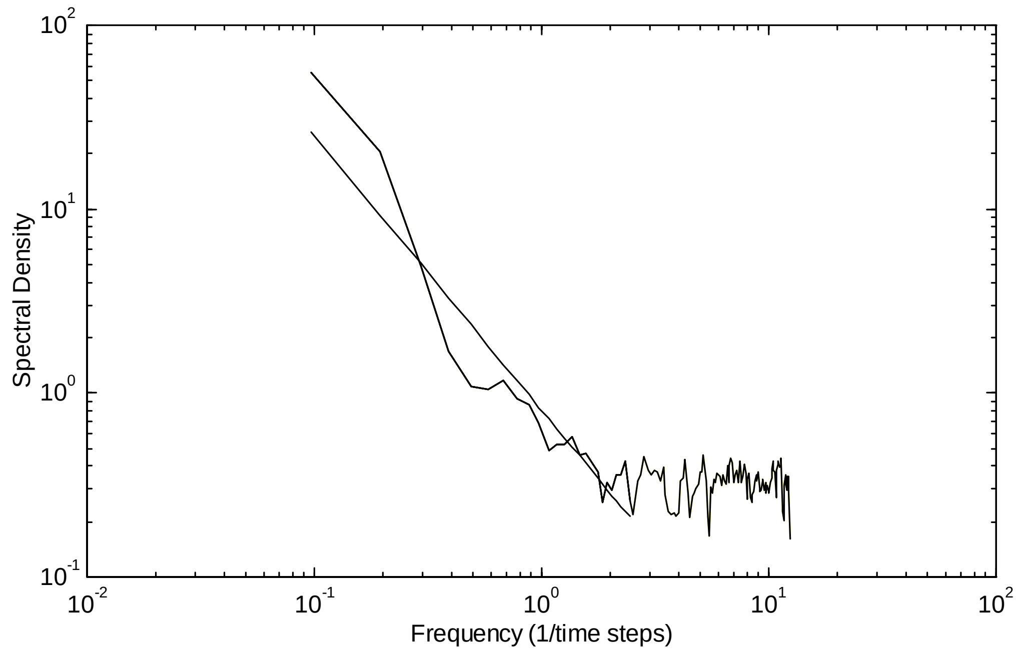

Figure 8: Power spectrum of temporal distribution of casualties. Left-hand side obeys a power law, the right-hand side behaves in a similar way to the turbulent dissipation "white noise" scale.

Conclusions

An alternative to the Lanchester equation, based on fractal dimensions, is suggested here and appears to usefully describe attrition. The approach assumes an inhomogeneous battlefield, with spatial and temporal correlations between entities. The correlations result from "local" personality rules.

The MANA model results appear to support equation 6 reasonably well. Equation 6 may be of great use in aggregated models of dispersed combat. Although the relationship between the fractal dimension of the distribution of the forces and the power law dependence of k is not clear from the results presented here, it is only necessary to understand how F changes as circumstances change, and how the function plotted in Figure 5 changes. In the case of this latter function, equation 7 is a poor approximation, so that it may be better to use the empirically obtained function in the figure. At the very least, the equations presented provide a convenient theoretical framework for understanding the outcome of dispersed confrontations.

Another important point is that data that display fractal characteristics often also display fat-tailed probability distributions. That appears to be the case for the results presented here. Importantly, the possession of power laws and fat-tailed distributions were identified as key requirements for combat to be able to be modeled as a statistically scaling system in Lauren 2000 [7].

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