Adjacency constraints in forest harvesting

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Abstract

Area-restricted adjacency constraints, which control the availability for harvest of specific blocks of trees relative to that of adjacent blocks, are a necessary part of many large-scale forest harvesting applications. However, such constraints are difficult to formulate and often precipitate major difficulties in the practical implementation of the solution algorithm. A new model which involves non-standard concepts is presented along with some test results from a case study.

1 Introduction to forest harvesting

A forest harvesting problem, FHP, is an application in which the present net worth of a forest is optimised subject to various strategic and site-specific constraints. Refer [3]. The trees in such a forest will belong to *croptypes* which are determined by species, seedling type and local growth factors such as soil and microclimate. The yield obtained from a given croptype depends solely on the year of harvest. For management purposes the trees are arranged in *blocks*. Usually all the trees in a block will be of the same croptype, and it is assumed that they will all be harvested in the one harvesting operation. The goal of the optimisation is to produce a yearly cutting plan which lists the blocks to be harvested each year for a time duration known as the tactical horizon.

The nature of the constraints will be varied. In a typical FHP application there may well be several thousand constraints. The most troublesome of these are those concerned with site-specific environmental aspects, as these require integer variables and are likely to be exponentially numerous. The adjacency constraints are of this type.

2 Clearfell regulations and adjacency constraints

In many countries there is a maximum clearfell area regulation, which specifies the maximum area of clearfell permitted. Once such a clearing has been made, none of the surrounding forest may be cut until the re-growth in the clearfell has attained what is called the *green-up period*. Such regulations are easy to legislate, but very difficult to incorporate into an FHP optimisation. As a consequence, forest managers in countries where such regulations are in force generally use various manual, LP or heuristic solution processes which deliver sub-optimal output.

Most of the existing literature is concerned with a simplified formulation of this adjacency problem. An assumption is made that if any two blocks are adjacent, then the harvesting of one of these will force the harvesting of the second block to be delayed until the green-up period has elapsed. This assumption greatly simplifies the problem as it ignores the significance of the actual size of each block. Murray [4] defines this type of formulation as a *unit restriction model*.

Various definitions are used to determine when two blocks are adjacent. Most frequently adjacent blocks are defined as having a common edge. This is the definition used in the practical example presented in Section 10. An alternative definition is to require only a common point. A more severe definition is to define two blocks as being adjacent if any point in one block is within say 100 metres of the second. This latter definition is particularly apt when the purpose of the adjacency constraints is to preserve wild-life corridors. Snyder and ReVelle [8] have made an interesting comparative study of these definitions in relation to an operational forest application. For the present model, any of these definitions may be selected by the user. The model merely requires a list of adjacent pairs of blocks to be available.

Particularly in hilly terrain, the size and shape of blocks may vary considerably. Where the block size is small, several adjacent blocks can often be harvested without exceeding the maximum clearfell area. In such a situation the problem of identifying violations of the clearfell regulations is non-trivial, as is the task of devising suitable adjacency constraints which will prevent such occurrences. Murray [4] defines this type of formulation as an *area restriction model*. It is possible to obtain a feasible solution to an area restriction application by using a unit restriction model, but such a solution would be sub-optimal as these unit restriction constraints would be too restrictive. The intention of this paper is to develop an area restriction model which can be solved by an optimisation method.

3 Literature survey

Nelson and Brodie [5] solve a small unit restriction application of 40 blocks over 3 time periods by listing every adjacency constraint explicitly and exhaustively like this:

$$x_{it} + x_{jt} \leq 1.$$

Here x_{it} is called a *block variable*. It takes value 1 if block i is harvested in year t, and takes value 0 otherwise. Meneghin, Kirby and Jones [2] follow a similar approach, but use ingenuity to aggregate constraints so that the single constraint

$$3x_{1,t} + 3x_{2,t} + 5x_{3,t} + x_{4,t} + x_{5,t} \leq 5,$$

represents a situation in which the block adjacencies are

$$(1,2),\ (1,3),\ (2,3),\ (3,4),\ (3,5).$$

In addition, various heuristic methods have been devised such as those by Sessions [7] and Clements [1]. Murray [4] has produced an insightful survey paper which concludes little progress has been made on the optimisation of area restriction models.

4 The concept of a maximal set

A *maximal set* is a set of blocks with the following properties. The total area of the blocks in the set is less than or equal to the maximum clearfell area. Each block in the set is linked to each other block in the set, perhaps transitively, by pair-wise adjacency relationships. If any one of the surrounding blocks were added to the set, then the total area would exceed the maximum clearfell area.

The set of blocks which are each adjacent to at least one block from a given maximal set, but not part of this set, is called the *perimeter set*. A trivial case occurs when an maximal set has no perimeter set. Such a case may be ignored. The algorithm which follows assumes that for every maximal set there is a unique perimeter set.

A *nuclear set*, is the result when we relax the requirement that the maximal set be maximal. In this case some, but not all, of the blocks in the perimeter set may be added to the nuclear set without exceeding the maximum clearfell area. Every nuclear set is a subset of some maximal set.

Both maximal and nuclear sets are strictly geographic concepts and carry no dependence on time or harvesting decisions.

5 How to find maximal and nuclear sets

This only needs to be done once. It is essentially an exhausive search based on the adjacency matrix with fathoming based on cumulative area so as to avoid any unnecessary searching. It is very similar to some of the search techniques used by David Ryan in his air crew scheduling work [6]. The present algorithm is designed to deal with maximal sets of up to 5 blocks. All necessary maximal and nuclear sets are identified prior to the first optimisation.

6 Selection of nuclear and maximal sets

Depending on the size of the blocks relative to the maximum clearfell area, there may be a very large number of nuclear and maximal sets which occur implicitly in an application. Fortunately, not all these sets are required. For this model only two types of nuclear sets need to be considered. First we include all possible nuclear sets comprising just one block. Secondly, we include all possible nuclear sets which comprise three blocks, with diameter 2 (ie block i is adjacent to j and j to k, but i is not adjacent to k). The inclusion of these nuclear sets allows the avoidance of many of the smaller maximal sets. In fact, no maximal set of size less than 5 needs to be explicitly included in the algorithm. There are several interesting theoretical results which support this model formulation. For example, if every possible nuclear set of size n is included, then no maximal sets of size n or $n+1$ are needed. The goal of the model formulation process is to obtain a relatively short list of maximal and nuclear sets which allows detection of all possible clearfell violations and permits the removal of these violations by the addition of appropriate adjacency constraints.

7 Clearfell violations with maximal and nuclear sets

Once the optimisation process has begun, clearfell violations are identified by scanning the block variables associated with every maximal and nuclear set in turn, timewise. If all the blocks in a set are harvested within the span of the green-up period, then the perimeter set is checked for any violations. This process is a key part of the solution algorithm. Success depends upon the fact that the number of maximal and nuclear sets involved are both relatively small, and that is why a limit was placed on the maximum number of blocks in a maximal set in Section 5 above. Each violation found will be specific to a given maximal or nuclear set and to a certain time interval.

8 Maximal and nuclear adjacency constraints

The definitions of an maximal set and its perimeter set permit a concise and highly aggregated formulation for the adjacency constraints. Let S be an maximal set, and P be the associated perimeter set. Let T be the length of the green-up period. Let s and p be the number of blocks in sets S and P respectively. Suppose that the blocks in set S are to be harvested between time periods t_a and t_b inclusive, with $t_b - t_a \leq T$. Then the required maximal adjacency constraint may be written

$$p \sum_{i \in S} \sum_{t=t_a}^{t_b} x_{it} + \sum_{j \in P} \sum_{t=t_b-T+1}^{t_a+T-1} x_{jt} \leq s \times p. \tag{8.1}$$

Similarly, suppose S is an nuclear set, and P is the associated perimeter set. Let a_i be the area of block i, A_S be the area of set S, A be the maximum clearfell area, and M be a suitably large constant. Then the required nuclear adjacency constraint may be written

$$M \sum_{i \in S} \sum_{t=t_a}^{t_b} a_i x_{it} + \sum_{j \in P} \sum_{t=t_b-T+1}^{t_a+T-1} a_j x_{jt} \leq A - A_S + M A_S. \tag{8.2}$$

The strategy during the solution algorithm is that only a very small number of these constraints will be included explicitly. For each violation found just one constraint will be included and the model then re-optimised. As seen in Section 7, each violation is specific to a particular maximal or nuclear set and to a specific time interval.

9 Aspects of the solution algorithm

Here is a formal summary of the algorithm.

Step 0 : First find all nuclear sets of size 1 and those of size 3 with dimension 2. Also find all maximal sets of size 5.

Step 1 : Solve the relaxed LP using column generation.

Step 2 : Test by scanning all maximal and nuclear sets obtained in step 0. If adjacency violations occur go to step 3; else go to step 4.

Step 3 : Add appropriate maximal and nuclear adjacency constraints. Go to step 1.

Step 4 : If fractional values of integer variables are present then go to step 5; else go to step 6.

Step 5 : Continue the branch and bound process by adding another constraint branch. Then go to step 1.

Step 6 : If this integer solution is within an acceptable interval then stop; else backtrack on the branch and bound tree. Return to step 1 (or stop if the tree is exhausted).

10 Results from a medium-sized NZ application

green-up period (years)	maximum clearfell area (ha)	objective value ($)	number of adjacency constraints	solution time (seconds)	number of maximal sets
2	40	102467313	102	163.7	89
3	40	102001440	109	222.9	89
4	40	101735249	194	182.0	89
5	40	101544893	84	152.5	89
6	40	101715870	51	135.0	89
2	50	102491074	57	190.0	72
3	50	102359179	55	141.7	72
4	50	102288085	55	154.8	72
5	50	102290770	51	126.4	72
6	50	102290770	33	118.3	72
2	60	102536555	50	127.6	75
3	60	102366762	50	128.0	75
4	60	102198570	57	135.1	75
5	60	102141168	56	114.4	75
6	60	102141168	36	108.1	75

Table 1: Output from Whangapoua Forest implementation

11 Conclusions

A working solution has been found to a long-standing problem in forest harvesting subject to maximum clearfell area regulations. Moderate and large applications may be solved in an optimization sense. The success of the method depends on the new concepts of the maximal set, the nuclear set and the associated perimeter sets. These concepts allow the effective checking of a relaxed solution for adjacency violations and the construction of highly aggregated constraints to remove these violations. In this way they guarantee that the final integer solution will completely satisfy the maximum clearfell area regulation. A workable solution has been developed and demonstrated with an operational case study. It is conceded that such an area restriction model would become intracticably complex if maximal sets consisting of large numbers of blocks were to occur. This is because the number of maximal and nuclear sets needing to be considered and individually checked for violations every iteration could increase exponentially. The recommended restriction of this model to applications in which the number of blocks in any maximal set is at most 5 is reasonable in an operational context. However, an application containing just a few instances of maximal sets of size greater than 5 is manageable. All of the maximal sets greater than 5 will need to be considered, along with any nuclear sets of size 5 or more lying in their interior. The best combination of methods that has been found is to use the new adjacency constraint formulation presented in this paper along with the column generation model for forest harvesting of McNaughton et al [3] and Ryan's constraint branching methods [6].

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