Optimization of Streamflow Estimation within the Framework of Conceptual Modeling

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Abstract

Traditionally, streamflow is estimated from a linear time-invariant basin response function with effective rainfall as input, which in turn is estimated from point rainfall observations using a runoff coefficient. With the advent of powerful computers this concept has been extended to gray-box type conceptual models. In this study the Variable Source Area (VSA) conceptual model has been investigated and its extension by Raper and Kuczera (1991) has been modified to apply to Little River catchment located west of Melbourne, Australia. Model parameters were first optimized using a combination of Gauss-Newton and steepest descent algorithms and later fitted to selected storms by the least squares method.

1.0 Introduction

Rainfall-runoff process in a watershed is too complex to be determined analytically, and therefore, simplistic procedures have been used by hydrologists for quite sometime. The linear time-invariant basin response function or the 'unit hydrograph' as it is often called, proposed by Sherman (1932) is still in use (Kachroo, 1992). However, its use is very limited because of the necessity to separate direct runoff from baseflow, which is fairly arbitrary. At present the most popular methods are based on conceptual rainfall-runoff models, which proliferated rapidly in the past three decades because of the availability of powerful computers. The type of conceptual model, most widely used, consists of a series of storage tanks interconnected by conduits, and flows governed by proportionate distribution among tanks representing surface detention, direct runoff, groundwater storage, etc. Another type, developed first by Hewlett and Hibbert (1963, 1967) is the variable source area (VSA) model. The latter type assumes that rapidly responding flow (quick flow) is mostly the precipitation falling on a contributing subwatershed within the topographically defined watershed, and not a percentage of rainfall falling over the entire watershed.

Raper and Kuczera (1991) simplified the VSA model by an inclined soil mass where a perched aquifer forms during precipitation. Formation of ephemeral perched aquifers have been observed in the field and reported by O'Loughlin (1981), Smith and Hebbert (1983), and Moore et al. (1986). There are several hypotheses about the causes of formation of such transient aquifers. One hypothesis is that initial storm bursts causes colloidal dispersion of clay particles which permeate downwards and clog the pores. Raper and Kuczera assumed that the perched aquifer behaves like a suspended mass of water in a container made from a permeable membrane. When the perched water table intersects the soil surface a seepage zone forms. Rainfall minus losses falling on the seepage zone manifests itself as quick flow.

Depletion from perched aquifer occurs either in the form of percolation or as interflow. Raper and Kuczera called their model CATPRO and applied it to estimate monthly flows of Salmon catchment in Western Australia. The author adopted CATPRO because of its demonstrated suitability in Australian conditions and modified it for use in estimating streamflow from a rainfall event. The author's model is described in the next section.

2.0 The Model

A schematic representation of the model is given in Fig. 1.

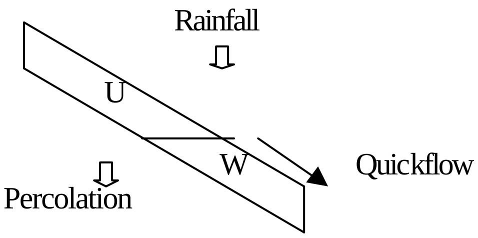

Fig. 1 Schematic representation of VSA conceptual model. U represents unsaturated zone and W is perched aquifer.

The components of the model are: (i) An inclined soil mass which is very long compared to its depth and has a water holding capacity of $S_{\max}$, (ii) S_t is the perched aquifer storage at any time t, (iii) Deep percolation from perched aquifer occurs at the rate D and is determined by multiplying the saturation length (length of W in the figure) at the bottom by a coefficient, C_d. Since the soil strip is assumed very thin, the saturation length at the bottom (L) and that at the top are assumed equal, (iv) The rain (P) falling on the soil mass in W undergoes no loss and flows as direct runoff and routed through a nonlinear reservoir with storage coefficient K and exponent m (RORB model of Laurenson and Mein, 1990), (v) P falling on unsaturated soil accretes with perched aquifer but part of it is lost which is estimated by multiplying the length of exposed soil by C_u and (vi) the relationship between L and S_t is given by $L = (S_t/S_{max})^e$, where exponent e is a parameter.

3.0 Case Study

The case study uses data from the 413 km^2 Little River catchment located west of Melbourne, Australia. Storm data is available for the catchment since 1965. The model was fitted to the data and the parameter values were optimized using NLFIT (Kuczera, 1987). NLFIT uses a combination of Gauss-Newton and steepest descent algorithms (Marquardt λ may vary between 0 and 2) and gives a multivariate normal probability distribution for the model parameters. The mean of the distribution provides the optimized parameters. Since the present model has seven parameters, viz., K, m, e, S_{max}, C_u, C_d, and S_t/S_{max}, it is possible to obtain innumerable sets of values of these seven parameters which would satisfy the least squares criterion of being optimum. The choice would narrow down considerably if only realistic values are considered.

Such model fitting exercise essentially consists of reproducing the peak and the basin lag or the time to peak. Since these vary from storm to storm, the calibration of parameters is dependent on storm characteristics, which has been known to hydrologists for a long time and thus pooling of parameters inferred from many storms is a standard practice. In this study different sets of pooled parameters which reproduced peak and basin lag of hydrographs well, were used to generate synthetic hydrographs. It was found, however, that different sets of parameters generated different shapes of hydrographs, though peaks and base lengths remained the same. This aspect was further investigated and possible influence of each parameter on the shape of the hydrograph was studied. Further investigation revealed that for each hydrograph, the value that each parameter assumes has a plausible explanation for the particular shape of the hydrograph. In other words, the shape of the hydrograph reflected what is to be expected when the value of a parameter was varied.

The normal expectations on hydrograph shape from model parameters are: the K parameter represents the delay time to peak and a higher value would cause slower rise or fall. When m is equal to 1, the conceptual reservoir is linear, and Teo (1988) studied 36 catchment hydrographs and found that m gets lower as drier the basin gets. The effects of the coefficients C_u and C_d can be explained from the studies of Ogawa et al. (1992), who noted that soil medium contains an intricate network of macropores, especially in the root-zone, which acts as hydraulic conduits and transmits water rapidly throughout the unsaturated columns so much so that there is no excess water flow as long as there is water stress. The e value of greater than unity gives a concave downward curve in L versus $S/S_{\max}$ plot and bears the simple explanation that some areas get saturated quickly and other areas require greater effort to get saturated. The converse is also possible, which is getting a considerable portion of a catchment saturated initially is difficult, but once that happens the rest of the area follow suit quickly. Therefore a knowledge of catchment geology is necessary to decide on an e value.

A large number of computer outputs was generated varying the value of each parameter and they were plotted to observe the trends and influences. The following general observations were made with the variation of parameters:

1. The peak discharge increases as $S_{\max}$ decreases.
2. The peak discharge decreases as K increases.
3. The basin lag decreases as m decreases.
4. As S_0/S_{max} increases, the initial flow value increases.
5. As exponent e decreases the peak increases and basin lag decreases.
6. Hydrograph peak and volume increase as C_u decreases.
7. The basin lag increases as C_d decreases.

It was observed that the shape of the hydrograph varied considerably with the degree of dryness or wetness of the basin. Thus it was felt that a faithful reproduction of the runoff characteristics should have a suite of hydrographs each representing different degree of wetness of the basin. With that objective in mind, four different storms, which occurred over the little river catchment were selected representing basin conditions from very dry to very wet.

In the next step, parameters were varied in accordance with the procedure outlined here so that the synthetic hydrograph shape represents the observed shape. This procedure involved dividing the base of the hydrograph into n intervals. For any interval i and for each of the seven variables, a directional derivative was constructed and its component in the vertical direction was determined, ie, a change in the direction of the arrow (see Fig. 2) for unit change of a given variable was computed by making two simulation runs with only changes made in one given variable. Let's call these X_{ij}, where $i = 0, \ldots, n$ and $j = 1, \ldots, 7$. Each X_{ij} was then multiplied with a weight a_j so that $a_j X_{ij}$ represented the total distance the model hydrograph had to move to reach the observed hydrograph at point i.

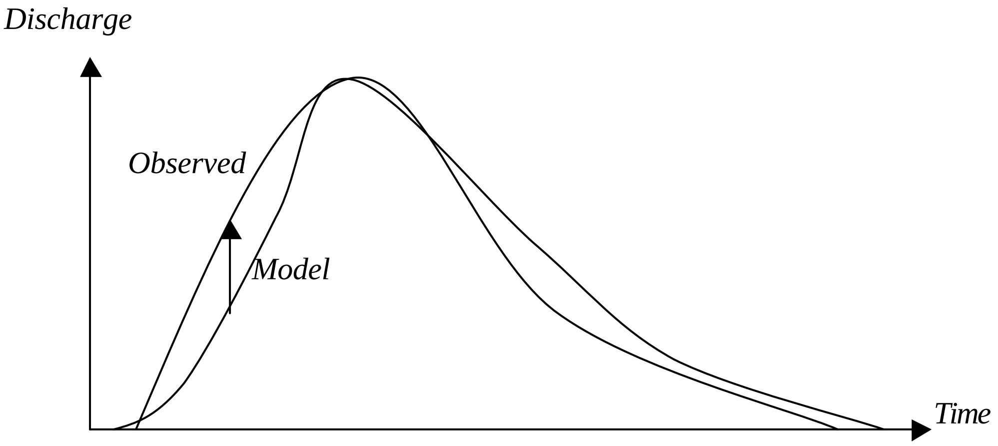

Fig. 2 Observed and model generated hydrographs. Arrow shows the direction in which model should be moved by varying the parameters.

Evidently, the weights would be different for different i points. The set of equations which needed to be solved to determine the optimum values of these weights are:

$$\sum_{j=1}^{7} a_j = 1$$

$$\sum_{j=1}^{7} a_j X_{ij} = y_i , \qquad i = 1, \ldots, n$$

where y_i is the difference between observed and model hydrograph values at point i. This optimization problem was handled by least squares method and the solution was obtained by

$$\hat{\mathbf{a}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$$

where $\hat{\mathbf{a}}$ is a vector of a values, $\mathbf{X}$ is a matrix of all the X_{ij} values and $\mathbf{Y}$ is a vector of y_i values.

4.0 Results and Discussion

The methods outlined above, which involve optimization at two levels within the framework of least squares estimation technique, were applied to four storms which occurred over the Little River catchment. Storm 1 occurred in February 1973 in very dry conditions, Storm 2 occurred in October 1976 in very wet conditions, Storm 3 occurred in November 1978 in relatively dry conditions, and Storm 4 occurred in October 1983 in relatively wet conditions. A list of the parameter values for all these storms computed using the afore-mentioned techniques are given in Table 1. Furthermore, simulation with these parameter values were carried out and it was found that the error in each case was less than 5 percent in the total volume of flow, which is an acceptable value in normal hydrological practices.

The four models can be applied to estimate the runoff from the watershed in the event of a storm. As to which model is appropriate at what time, a knowledge of moisture content of the basin is necessary. This can be done using a soil-moisture accounting model, for example, the CRAE model of Morton (1985).

Table 1 Parameter values of the model for four storms on Little River catchment.

Parameter	Storm 1	Storm 2	Storm 3	Storm 4
K	4.2	2.0	4.0	4.0
m	0.62	0.95	0.71	0.71
e	1.5	1.5	1.5	1.5
S_{max}	44	44	44	44
C_u	0.30	0.02	0.24	0.1
C_d	0.80	0.01	0.80	0.1
S_o/S_{max}	0.0	0.31	0.14	0.19

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