Bookmobile routing and scheduling in Buskerud County, Norway

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Abstract

A bookmobile is a specially adapted bus or van used as part of the outreach operations of public library systems. Bookmobiles play a significant part in the service of the public library system in Buskerud County, Norway. They are used to deliver and collect library materials (printed books, audio books, periodicals, and music) to and from lender groups throughout the County, many in remote areas. The question of how best to utilize the County's bookmobile resources can be modelled as an interesting variation of one of the classical models of OR; the vehicle scheduling and routing problem. The combination of the features that make this scenario non-standard include: multiple depots, simultaneous cost minimization and prize collection objectives, differing customer service levels, time windows, route start time flexibility for some routes, multiple route duration restrictions, route lunch breaks, and overnight stays on certain routes. We report on a model for the bookmobile problem and the outcome of applying it to the Buskerud County bookmobile system.

Key Words: Bookmobile problem, vehicle scheduling, integer programming, case study.

1. Introduction

Buskerud County, Norway, is located to the west of Oslo. The public libraries of the County, based in the towns of Drammen and Gol, both operate bookmobiles to serve identified groups of its lenders that cannot visit a library. These bookmobiles, which are specially designed vans, carry various library materials, such as: printed books, audio books, periodicals, and music, to road lay-bys and private farmhouses, in remote parts of the County, and to schools and other libraries. The Buskerud Library Department is under considerable financial pressure to make efficient and effective use of its bookmobile operations. This led to us being invited by those responsible for the bookmobile system to examine their operations, with view to making suggestions for improvement. It became clear to us that the question of how to best utilize the County's bookmobile resources can be modelled as an interesting variation of one of the classical models of OR; the vehicle routing and scheduling problem (VRSP). In this problem, a number of vehicles, subject to capacity restrictions, must visit and service all of a given set of customers at known locations, at minimum total cost. We now review some of the relevant (VRSP) literature.

Early surveys of the original vehicle routing problem were given by Ghare, Turner and Foulds [10], and Foulds and Watson-Gandy [9]. Since then numerous refinements of the basic model, including the addition of time windows (eg Potvin, Kervahut, Garcia and Rousseau [13]), and time-dependent travel times (eg Malandraki and Daskin [12]), have been reported. Solution techniques for the VRSP usually involve either integer programming, including column generation (eg Ryan, Hjorring, and Glover [14], Desrochers, Desrosiers, and Soloman [7], and Desrosiers, Dumas, Soloman, and Soumis [8]) or heuristics (eg Gillett and Miller [11] and Taillard, Badeau, Gendreau, Guertin, and Potvin [15]). A typical VRSP case study, involving the practical use of some of the above models and techniques, has been reported by Basnet, Foulds, and Igbaria [1]. More recently, there has been interest in what has come to be known as the vehicle routing-allocation problem (VRAP). This model is similar to the VRSP, except that it may not be feasible to service all customers, and part of the problem is to decide which customers to service. (See Beasley and Nascimento [3] for a framework.) Both algorithms (eg Butt and Ryan [5]), and heuristics (eg Chao, Golden, and Wasil [6] and Beale [2]) have been posed for this and related problems. However, while the bookmobile problems that we are interested in are similar to the VRAP, they have a special combination of features that make them significantly different from any of the models that we have found in the VRSP literature. These include: multiple depots, simultaneous cost minimization and prize collection objectives, differing customer service levels, time windows, route start time flexibility for some routes, multiple route duration restrictions, route lunch breaks, and overnight stays on certain routes. The only reported work on lunch breaks that we could find is the compact IP model of employee scheduling devised by Brusco and Jacobs [4]. However this model has little to do with the VRSP.

In the next section we describe a special case of the County's bookmobile system, a model of its operations, and the outcome when we applied our approach to the actual problem at hand. In Section 3 we generalize the description of the operation of Section 2 to more general scenarios. We end the paper with conclusions drawn from this case study and suggested directions for further research.

2. The Drammen Bookmobile System

The first, and most pressing, question that we were asked to examine was how to improve the bookmobile service to a selected number of lender locations surrounding the town of Drammen.

2.1. **A Description of the Drammen Operation.** A single bookmobile is operated, once per week, over a four-weekly cycle. It must service certain lender locations (termed 'compulsory lenders') once every fourteen days (ie twice), but at the same time of day. This is because some of the boroughs that comprise the County pay for certain lenders located within them to receive this service. There is also another type of lender (termed 'optional lenders') each of which may be visited at most once during the cycle. The optional lenders are private individuals whom it is desirable, but not strictly necessary, to visit. The regime of identified routes (termed 'runs') is carried out once, in the same way, during each time cycle. The primary issue is one of devising a set of feasible runs for the bookmobile so that the maximum number of (optional), unweighted lender locations are visited. The secondary issue is one of achieving this at minimal cost. We now go on to describe the operation in more detail, and to highlight factors that constrain the operation.

Each run begins at Drammen, visits a given sequence of lenders that it services in turn, and finally returns to Drammen. In this sense, Drammen is a unique 'depot' in terms of the VRSP. The locations of all lenders are known, as is the (season-dependent) travel time for the bookmobile to traverse all feasible road segments that link them. (Because of the terrain of the County, the road network is somewhat sparse.) We can assume, without loss of generality, a given set of road segment driving times for a given season of the year. Because it is assumed by the planners that costs are directly proportional to time, the secondary issue, mentioned above, reduces to one of minimizing the total elapsed time (travel, service, break, and idle time, combined over all runs).

Because the capacity of the bookmobile is ample to service any combination of lenders on any run, the vehicle is not 'capacitated' as in the VRSP. In this sense our problem is more akin to the multiple travelling salesman problem, than to the VRSP. If any lender is visited at all, its servicing takes a known time (termed its 'duration'). Moreover, each lender can be serviced only during a known time window, that remains constant for the lender throughout the cycle. Servicing starts immediately upon arrival. Suppose that the time windows of two lenders who are visited one immediately after the other, on a run, are such that idle time is necessary. The idle time must occur just before the departure from the earlier lender. Furthermore, none of the runs can exceed a given number of time periods, there is a restriction on the average time of all runs, and each run must contain a continuous break that must overlap with the time interval from three time periods before the midpoint (in terms of time) to three time periods after the midpoint. Each break takes place at a lender, immediately after servicing that lender. Finally, the start time of each run is arbitrary.

2.2. **A Model of the Drammen Operation.** We now create a model of the operation that has just been described. To this end we first introduce the necessary notation.

Basic dimensions. Let:

n**:** the number of locations. (The depot, being the town of Drammen, is denoted by location 1. The locations of the compulsory lenders are denoted by locations: $2, 3, \dots, q$. The locations of the optional lenders are denoted by locations: $q+1, q+2, \dots, n$; where $1 < q < n$.)
m**:** the given number of runs to be carried out in each time cycle.
k**:** the index of the run carried out in the kth week of the time cycle.

Input data. Let:

M, N**:** relatively large, given numbers,
d_i**:** duration of lender i,
a_i**:** the earliest time period during which the servicing of lender i can begin,
b_i**:** the latest time period during which the servicing of lender i can begin. (The interval $[a_i, b_i]$ represents the time window during which the servicing of lender i must begin, if it takes place at all.)
t_{ij}**:** travel time in proceeding directly from location i to location j (For technical reasons it is assumed that all $t_{ij} > 0$. The triangle inequalities $t_{ij} \leq t_{ih} + t_{hj}$, for all i, h, j do not have to be satisfied unless column generation is used.)
u**:** the length of the break on each run, expressed as a number of time periods,
P**:** maximum allowable number of time periods for any run, and
T**:** maximum average number of time periods over all runs of a complete cycle.

Decision variables. Let:

s_k**:** the start time of the kth run,

$$v_{ik}\text{:} \begin{cases} 1 & \text{if there is a break on the } k\text{th run, immediately after servicing lender } i \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijk}\text{:} \begin{cases} 1 & \text{if the } k\text{th run proceeds directly from location } i \text{ to location } j \\ 0 & \text{otherwise} \end{cases}$$

y_{ik}: the arrival time of the kth run at location i. (If the kth run does not visit location i, y_{ik} is defined arbitrarily.)

z_j^o: $\begin{cases} 1 & \text{if compulsory lender } j \text{ is visited in weeks 1 and 3} \\ 0 & \text{otherwise} \end{cases}$

z_j^e: $\begin{cases} 1 & \text{if compulsory lender } j \text{ is visited in weeks 2 and 4} \\ 0 & \text{otherwise} \end{cases}$

We now develop a model of the Drammen operation. The primary objective is to maximize the number of optional lenders that are serviced. The secondary objective is to carry out the primary objective in the minimum feasible total elapsed time.

(1) $$\max \left(M \sum_{i=q+1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} x_{ijk} \right) - \sum_{k=1}^{m} (y_{1k} - s_k)$$

Subject to:

Each lender can be visited at most once on any given run:

(2) $$\sum_{i=1}^{n} x_{ijk} \leq 1 \qquad j = 2, 3, \ldots, n; \quad k = 1, 2, \ldots, m$$

Each optional lender is visited at most once:

(3) $$\sum_{i=1}^{n} \sum_{k=1}^{m} x_{ijk} \leq 1 \qquad j = q+1, q+2, \ldots, n$$

Each compulsory lender must be visited every fourteen days:

(4) $$\left. \begin{aligned} \sum_{i=1}^{n} \sum_{k=1,3} x_{ijk} &= 2z_j^o \\ \sum_{i=1}^{n} \sum_{k=2,4} x_{ijk} &= 2z_j^e \\ z_j^o + z_j^e &= 1 \end{aligned} \right\} \qquad j = 2, 3, \ldots, q$$

Each run must depart from Drammen:

(5) $$\sum_{j=2}^{n} x_{1jk} = 1 \qquad k = 1, 2, \ldots, m$$

Each run must return to Drammen:

(6) $$\sum_{i=2}^{n} x_{i1k} = 1 \qquad k = 1, 2, \ldots, m$$

If a run arrives at a lender, it must leave that lender:

$$\sum_{i=1}^{n} x_{ihk} = \sum_{j=1}^{n} x_{hjk} \qquad h = 2, 3, \ldots, n; \quad k = 1, 2, \ldots, m \tag{7}$$

Arrival times must account for duration, travel, and break times:

$$\begin{aligned} & y_{ik} + d_i + uv_{ik} + t_{ij} + N(x_{ijk} - 1) \le y_{jk} \\ & \qquad\qquad i = 2, 3, \ldots, n; \quad j = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, m \\ & s_k + t_{1j} + N(x_{1jk} - 1) \le y_{jk} \qquad j = 2, 3, \ldots, n; \quad k = 1, 2, \ldots, m \end{aligned} \tag{8}$$

Each run has exactly one break:

$$\sum_{i=2}^{n} v_{ik} = 1 \qquad k = 1, 2, \ldots, m \tag{9}$$

If the break on the kth run occurs at lender i, then the kth run must service lender i:

$$\sum_{j=1}^{n} x_{ijk} \ge v_{ik} \qquad i = 2, 3, \ldots, n; \quad k = 1, 2, \ldots, m \tag{10}$$

The servicing of a lender must occur during the lender's time window:

$$a_i \le y_{ik} \le b_i \qquad i = 2, 3, \ldots, n; \quad k = 1, 2, \ldots, m \tag{11}$$

There is a time limit on the duration of each run:

$$y_{1k} - s_k \le P \qquad k = 1, 2, \ldots, m \tag{12}$$

There is a time limit on the average duration of all runs:

$$\sum_{k=1}^{m} (y_{1k} - s_k) \le mT \tag{13}$$

Part of the break of each run must occur approximately half-way through the run:

$$\begin{aligned} & y_{ik} + d_i \le \frac{1}{2}(y_{1k} + s_k) + 3 + N(1 - v_{ik}) \\ & \frac{1}{2}(y_{1k} + s_k) - 3 \le y_{ik} + d_i + u + N(1 - v_{ik}) \\ & \qquad i = 2, 3, \ldots, n; \quad k = 1, 2, \ldots, m \end{aligned} \tag{14}$$

Each compulsory lender must be visited at the same time of day on both of its runs:

$$\left.\begin{aligned} y_{i1} &= y_{i3} \\ y_{i2} &= y_{i4} \end{aligned}\right\} \qquad i = 2, 3, \ldots, q \tag{15}$$

And, finally, simple constraints:

$$\left.\begin{aligned} x_{ijk} &\in \{0,1\} \\ v_{ik} &\in \{0,1\} \\ z_j^o, z_j^e &\in \{0,1\} \\ y_{ik} &\in \{1,2,3,\dots\} \\ s_k &\in \{1,2,3,\dots\} \end{aligned}\right\} \qquad \begin{aligned} i &= 1,2,\dots,n \\ j &= 1,2,\dots,n \\ k &= 1,2,\dots,m \end{aligned} \tag{16}$$

2.3. **Application of the model.** We now report on the outcome when the numerical instance for the Drammen system corresponding to Model 1 was solved. The input data are given in Table 1, with 156 time periods, of 5 minutes each, defining the maximum length of any working day. Period 0 begins at 8.00 am, period 1 begins at 8.05 am, and so on. Also, $n = 22$, $q = 18$, $m = 4$, $M = 1000$, $N = 100$, $u = 8$, $P = 108$, and $T = 96$. Lenders 9 and 15 are actually the same, except for their time windows. This means that there is one special lender that is visited four times, that is, alternating in the afternoons and in the mornings, week by week. This factor is reflected in the following specialized version of constraint (4):

$$\begin{aligned} \sum_{j=1}^{n} x_{9jk} &= 1 \qquad k = 1,3 \\ \sum_{j=1}^{n} x_{15,jk} &= 1 \qquad k = 2,4 \end{aligned} \tag{17}$$

TABLE 1. The Input Data for the Drammen System

	2	3	4	5	6	7	8	9	10	11	12
d_i	12	3	4	4	4	4	3	6	6	4	4
a_i	6	1	48	72	84	96	108	96	108	12	12
b_i	42	153	96	154	154	153	150	150	152	152	152

	13	14	15	16	17	18	19	20	21	22	
d_i	4	4	6	4	4	4	4	4	4	4	
a_i	24	12	12	12	12	12	96	1	1	48	
b_i	96	152	152	152	152	152	152	152	152	152	

Even given the relatively modest dimensions of this instance, it could not be solved to optimality using a standard IP software system, namely XPRESS-MP [16]. The best solution obtained produced the following runs, with the sequence of lenders serviced (starting and ending at location 1), as indicated by the order of the columns, as shown in Tables 2-5. Table entries represent time period numbers when each event begins, including the break.

Table 2. The run for the first week (Duration = 87)

Location	1	2	18	17	15	11	12	5	6	13	4	1
Arrive		20	34	39	46	58	72	78	84	90	96	103
Break						62						
Depart	16	32	38	43	52	70	76	82	88	94	100	

Table 3. The run for the second week (Duration = 52)

Location	1	14	3	16	10	9	8	7	1
Arrive		94	99	103	108	124	132	137	144
Break					114				
Depart	92	98	102	107	122	130	135	141	

Table 4. The run for the third week (Duration = 106)

Location	1	2	18	17	15	11	12	5	6	13	4	19	20	22	1
Arrive		20	34	39	46	58	72	78	84	90	96	101	109	114	122
Break						62									
Depart	16	32	38	43	52	70	76	82	88	94	100	105	113	118	

Table 5. The run for the fourth week (Duration = 56)

Location	1	21	14	3	16	10	9	8	7	1
Arrive		89	94	99	103	108	124	132	137	144
Break						114				
Depart	88	93	98	102	107	122	130	135	141	

The combination of the above four runs provides an objective function value of $(1000)4-87-52-106-56 = 3699$, with all four optional lenders serviced, and a total elapsed time, for all four runs, of 301 time periods. These runs represent the following improvements to the Drammen system. For the first time, the optional lenders are serviced. Indeed all four of them are serviced. Also, the total elapsed time is 17% less than the schedule that was originally in use. That original schedule was not only longer, but did not include any of the four optional lenders. The runs shown in Tables 2–5 were accepted by the Drammen librarians, and have become the modus operandi for Drammen. We now go on to describe larger problems.

3. Operations with Larger Numerical Instances

We now briefly describe extensions of the Drammen operation, discussed in the last section, to the routing and scheduling of the bookmobiles that serve firstly, the southern part of Buskerud County, and secondly, the whole County. As will be seen, these operation are significantly more complex than the Drammen operation, both in terms of constraints and size.

3.1. **A Description of the Southern Buskerud County Operation.** The Southern Buskerud County operation has most of the same features and constraints as the previously mentioned operation, except for the following factors. Most of the lenders are compulsory. Most of the cumpulsory lenders must be serviced exactly once during each time cycle. The rest of the compulsory lenders must be serviced exactly twice, as in the Drammen operation. But as we now have more than one run per week, this requirement must now be reflected in a specific constraint in any model of the operation. However each of the lenders that must be visited twice must not only be visited at the same time of day, as before, but also on the same day of the week (eg every second Thursday at 11.00 am). This last restriction is now important, as there are to be fourteen runs per four-weekly time cycle, as opposed to one per week, as in the Drammen case. There are no lenders that must be visited more than twice. There are also a few optional lenders, that as before, can be visited at most once.

However the main difference from the Drammen operation comes about due to the fact that some of the lenders are located in remote areas, far from the depot, which is still the town of Drammen. This makes it necessary to incorporate an overnight stay (of a single night) at a certain hotel, at a given location, into one of the runs. This means that most of the runs are completed within one working day, as before, but one run is completed in exactly two working days. Certain identified lenders must be serviced by this unique, two-day run. However, there is time available on that run to service additional lenders, that may be compulsory or optional. The question of which, if any, additional lenders to service on the hotel run is part of the decision problem. Naturally, for the two-day run, the hotel departure time, and all subsequent times on the run, must be reset with respect to the hotel arrival time. Once again, each working day must have a break during the middle of its elapsed time. Hence the two-day run has two breaks. The above imply that, in terms of VRSP terminology, the Southern Buskerud County operation is a VRAP with additonal, compulsory lenders requiring differing service levels, and also with one run being up to twice the normal allowable duration. Also, the numerical instance corresponding to the this latter operation is far bigger than that for the Drammen operation. We now describe the complete bookmobile operation for the whole of Buskerud County.

3.2. **A Description of the County-wide Operation.** The County-wide operation differs from the previous operation in the following ways. The operation is based at two towns, Drammen and Gol, that serve as depots. That is, each bookmobile is based at exactly one of the towns, in the sense that it begins and ends all of its runs at one town. The number of runs based in each town is given. However, the question of which depot will service each lender is part of the decision problem. Also, in addition to the known hotel with its dedicated lenders serviced out of Drammen, there are to be four overnight runs based in Gol. There are a number of available hotels, out of which these four must be chosen.

4. Conclusions and Suggested Directions for Further Research

There do not seem to be any models or solution techniques for bookmobile routing and scheduling reported in the open literature. This version of the VRAP is complicated by a unique combination of factors. We have reported the outcome of a successful implementation of standard integer programming model for a small, practical scenario. However, it is likely that more effective techniques will have to be employed to produce useful schedules for instances of the dimensions of the Southern Buskerud and County-wide scenarios. To this end, we hope to look into the use of column generation, with the capital budgeting problem as the basic IP model to be relaxed. We plan to publish the results of that investigation elsewhere.

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References

[1] Basnet, C.B., Foulds, L.R., and Igbaria, M. FleetManager: a microcomputer-based DSS for vehicle routing. Decision Support Systems 16(1996)195-207.

[2] Beale, I.R. Subset selection routing: modelling and practicalities. Unpublished doctoral dissertation, School of Information Science, Massey University, New Zealand, 2001.

[3] Beasley, J.E., and Nascimento, E.M., The vehicle routing-allocation problem: a unifying framework. TOP 4(1996)65-86.

[4] Brusco, M.J., and Jacobs, L.J. Optimal models for meal-break and start-time flexibility in continuous tour scheduling. Management Science 46(2000)1630-1641.

[5] Butt, S.E., and Ryan, D.M. An optimal solution procedure for the multiple tour maximum collection problem using column generation. Computers and Operations Research 26(1999)427-441.

[6] Chao, I.M., Golden, B.M., and Wasil, E.A. The team orienteering problem. European Journal of Operational Research 3(1996)464-474.

[7] Desroschers, M.J., Desrosiers, J., and Soloman, M. A new optimization algorithm for the vehicle routing problem with time windows. Operations Research 40(1929)342-354.

[8] Desrosiers, J.,Dumas, M., Soloman, M., ans Soumis, F. Time constrained routing and schedulimg. In Handbooks in Operations Research and Management Science, Vol. 8: Network Routing. Ball, M.O., Magnanti, M.L., and Nemhauser, G.L., (eds). North-Holland, Amsterdam, The Netherlands (1995).

[9] Foulds, L.R., and Watson-Gandy, C.D.C. Vehicle scheduling: a survey. New Zealand Operational Research 9(1981)73-92.

[10] Ghare, P.M., Turner, W.C., and Foulds, L.R., Transportation routing problem: a survey. AIIE Transactions 6(1974)288-301.

[11] Gillett, B., and Miller, L. A heuristic algorithm for the vehicle dispatching problem. Operations Research 22(1974)340-349.

[12] Malandraki, C., and Daskin, M.S. Time-dependent vehicle routing problems-formulations, properties and heuristic algorithms. Transportation Science 26(1992)185-200.

[13] Potvin, J.-Y., Kervahut, T., Garcia, B.-L., and Rousseau, J.-M. The vehicle routing problem with time windows. Part I: Tabu search. INFORMS Journal on Computing 8(1996)158-164.

[14] Ryan, D.M., Hjorring, C., and Glover, F. Extensions of the petal method for vehicle routing. Journal of the Operational Research Society 44(1993)289-296.

[15] Taillard, E., Badeau, P., Gendreau, M., Guertin, F., and Potvin, J.-Y. A tabu search heuristic for the vehicle routing problem with soft time windows. Transportation Science 31(1997)170-186. 1

[16] XPRESS-MP, Dash Optimization, Blisworth NN73BX, UK.