Nonlinear Programming and Risk Management

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Abstract

This paper presents two applications of nonlinear programming to risk minimisation in finance. The first addresses the issue of hedging using either options or futures, the proportion of which can be optimised with a nonlinear programming model. The second application is the use of nonlinear programming to minimise risk in asset allocation with the additional capability of artificial intelligence. The model focuses on achieving a target level of return while minimising downside risk The use of artificial intelligence is to indicate the possibility of derivatives being used to cushion downside movement, in particular put options on the physical asset.

1. Balancing Options and Futures

The first application to be discussed is the use of optimisation to find an appropriate balance between options and futures. Given the need to hedge, the question remains what proportion should be in each, and how much should be left unhedged. Suppose a firm wishes to minimise the domestic currency cost of repaying a foreign currency debt at some time in the future. If the firm expected the exchange rate to be much higher than the than that quoted in the futures market, then the firm could purchase a deep out of the money call option. It would not cost very much, but then it would not get exercised until the rate was low. It would be very similar to being unhedged, yet there would be a safety net in case there is a sharp fall in the exchange rate. A symmetrically opposite scenario could apply in the case of foreign currency receivables and the expectation is that the exchange rate would be much lower than that quoted in the futures market; a deep out of the money put option could be purchased.

The objective function for the first case would be an expected value calculation of the cost in domestic currency of repaying the foreign currency debt. An integration would proceed in two parts for the proportion hedged with options: integration of the exchange rate multiplied by its probability density function up to the strike price, then integration of the density function times the strike price beyond that level. Also included in the objective function is the cost of the call function, the cost incurred using a forward rate for the proportion hedged using futures, and the expected cost for the proportion left unhedged using a forecasted expected value for the actual rate at the future time period.

Constraints in the model would consist of upper and lower bounds on the proportions hedged either way (or left unhedged); for example many resource-based companies have a fixed proportion left unhedged as company policy. There would also be a balancing equation summing the individual proportions to unity.

A further constraint is required in order to place a limitation on the risk involved. The firm would wish to place a low probability that the actual cost was greater than some small percentage increase over the riskless cost; say in the range 0.5-10%. This takes the form of a chance-constraint, where the probability level chosen (around say 5-10%) forms the right hand side term and the expression on the left hand side represents the random variation with an assumed probability distribution.

There would be a similar optimization for foreign funds receivable, with the same terms using a put option and the objective of maximising nett expected revenue in domestic currency.

2. Options and Futures Implementation

The model has been developed and implemented in a Windows PC environment, using an Excel spreadsheet as the user interface. The optimization code used is AESOP, an extension of the well-known MINOS code of Murtagh and Saunders (1987) to include mixed-integer nonlinear programming and advanced heuristics to speed convergence (Murtagh and Sugden, 1994). The implementation performs the optimisation calculation for a range of possible future exchange rates, producing a graphical representation of the changes in proportions optimized. It takes less than 1 minute to produce a 20-point scan of possible future exchange rates.

Calculation of the option price is undertaken with an adaptation of the Black-Scholes option pricing formula to allow for both interest rate and exchange rate variations between currencies. Calculation of the expected value terms in the objective function is undertaken using numerical integration, and calculation of the chance-constraint expressions and the option price requires determination of the cumulative normal distribution function. A polynomial approximation formula available in the handbook by Abramowitz and Stegun (1986) is used for this.

The results obtained in experimentation with both the Japanese Yen and the United States Dollar indicate a high level of sensitivity to the required confidence level, i.e. probability requirement. There was a range of possible currency outcomes for which a combination of all three proportions was non-zero, except in the case for small allowed increase over the riskless cost.

3. Risk measures in asset allocation

The second application of nonlinear programming is to consider the issue of asset allocation and downside risk. To devise the best means of asset allocation among a number of sectors it is necessary to examine the patterns of growth of the accumulation indices in each sector and the variability in their performance; optimise the allocation of assets to achieve a measured balance between risk and return, and consider the use of derivatives to cushion any downside movement that does occur.

The accumulation indices are analyzed to build up a probability distribution of returns for each sector from the raw data. Also an appropriate forecasting model is developed to predict next-period returns.

Given the above information the optimisation task is to allocate assets to sectors to achieve a target level of return for the whole portfolio at minimum risk. Risk can be defined in many ways; in this paper the preferred representation is one of downside risk i.e. minimizing the likelihood of downside movement from expected value.

Determination of when to buy options and at what strike price relies on elements of artificial intelligence and fuzzy set theory. In choosing a sector with an acceptably high expected return we would also choose to hedge with put options if the accumulation index displayed a significant probability of downward movement below the strike price.

3.1 Distribution of Returns

The raw data is in the form of accumulation indices reflecting capital gain and dividend reinvestment. Returns are measured by the relative movement of indices over each time period. The previous 52 time periods up until present time are measured.

Rather than calculate the mean and standard deviation and assume a normal distribution, the actual distribution of returns is modelled. The variation from expected value is measured for each of the previous time periods and weighted exponentially in time to give maximum credence to recent time periods and minimal to the oldest. Each of these measures forms a separate constraint in the model.

In determining the likely performance of individual sectors for the next period in the future a state-space vector version of an auto-regressive integrated moving average forecasting technique is used (Dunsmuir & Murtagh, 1993, Ostermark, 1991).

3.2 Minimisation of Risk

The most widely used measure of risk in asset allocation is variance, which is well known and forms the basis of the Capital Asset Pricing Model. As techniques of numerical optimization have improved, it has been applied to quite large portfolio analysis problems

(Markowitz, 1987, Perold, 1984). However there are two fundamental difficulties with the mean-variance model:

- The assumption that returns are normally distributed about the mean, and the required storage and calculation of a (usually dense) variance co-variance matrix.
- Variance being used as a measure of risk equally penalises both upside and downside variation.

Other measures of risk have appeared in recent literature, including mean absolute deviation which does not require an assumption of normality (Konno and Yumazaki, 1991, Speranza, 1993, Zenios and Kang, 1992).

The focus of this paper is on downside risk, so the preferred risk measure is the negative semi-absolute deviation, which only measures the under-performance from expected value. A similar concept using variance would be the negative semi-variance and it is possible to choose this as a measure of risk as an option in the model.

The effect of using this approach is that we are minimising the probability of downside movement from expected value. The optimisation process will favour those sectors with a history of low downside risk (as measured by their actual probability distributions).

However minimising risk is insufficient in itself and would result in low-yield sectors being selected. It is necessary to specify a target level of return so that the model chooses the combination of sectors that achieve the target level of return with lowest downside risk.

Two aspects of artificial intelligence are implemented in the current system: expert systems and machine learning, using fuzzy set theory (Tapia and Murtagh, 1991,1992). This approach was used in the current application to determine the following:

- Which sectors to hedge with derivatives, specifically put options on the physical positions.
- The strike price of the put options.

The two requirements are bound up together; for example in comparing two sectors for inclusion in the allocation one may have a modest expected return and low downside risk, while the other may have a high expected return but also significant downside risk. The machine learning process estimates the probability of downside movement, and if it is above a threshold value then if the sector is included in the allocation then also a corresponding put option is purchased.

The data required for derivation of the options prices is the volatility; the Black-Scholes formula is used for calculation of actual prices (Black and Scholes, 1973).Note that the capability of hedging the physical position with put options is available as an additional feature in the implementation of the model; the capability can be switched off if so desired.

4. Asset Allocation Implementation

The implementation of the model minimises risk subject to a target level of return. Other constraints in the model include the balancing equations and user-specified limitations on the selection of asset classes.

This model was also implemented using the AESOP nonlinear integer code. Using nonlinear integer programming it is possible to enforce practical considerations, including minimum transaction size, transaction cost and incremental marketable quantities. Upper and lower bounds and bounds on individual sectors can be enforced, as also can proportionality requirements of asset classes within the total mix.

The data required to run the model are details of the previous history of each sector, in the form of period-end value of the accumulation index. This data is analysed to provide both the probability distribution of returns and forecasts of future performance. If desired user-supplied forecasts of returns in each sector can be specified.

There are optional measures of risk which can be specified; minimise negative semi-absolute deviation, negative semi-variance, or variance. Experimentation with each of the three indicates that negative semi-absolute deviation does indeed produce the best performance.

Periodic adjustment of the asset mix incurs transaction fees on both positive and negative adjustments, and upper limits on the extent of adjustment in each sector can be imposed.

The following may be specified for each sector by the user:

- Previous period prior allocation
- Previous period options position
- Current period expected return (optional)
- Lower limit for sector in total mix (default 0%)
- Upper limit for sector in total mix (default 60%)
- Maximum adjustment in any one period (default 60%)
- Transaction cost (default 0.8%)
- Options volatility, for calculating options price (default 20%)
- Maximum hedging of physical position (default 100%)

The system is implemented in a WINDOWS-based PC environment with very rapid solution times. The user interface is via a wizard-style sequence of screens (using Visual Basic) in which selections and data may be adjusted as required. .The output report also incorporates the Crystal Reports™ engine for professional reporting and graphical display.

The system has been applied to several specific sets of asset classes for the countries:

- Australia
- Japan
- Malaysia
- Singapore

There is also a version which optimises asset allocation amongst 32 different countries, based on the MSCI accumulation index for each.

Experimentation with the model over a range of back-testing periods has demonstrated that negative semi-absolute deviation is the risk measure in the objective function that achieves the most stable results for all levels of target returns. The 32-country international version in particular has enjoyed success in achieving consistent results.

5. Conclusions

Computer implementation of both models has demonstrated their effectiveness in balancing risk and return. For the first application, the model has shown to be useful in testing the sensitivities of the options and futures balance to levels of uncertainty in exchange rate prediction and also to risk tolerance, as reflected by allowed probabilities and allowed margins for exceeding the risk-free cost.

The results obtained for the second application have demonstrated to be useful for asset allocation and long-term investment decisions. Perhaps as importantly, the sector allocations made by the model display relatively low variability themselves, although this is dependent on the limits on each sector defined by the user and the target level of return specified.

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