Piecewise Linear Offers in the New Zealand Electricity Market

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Abstract

The New Zealand Electricity Market currently accepts offers from suppliers in the format of a step function. This study aims to investigate the effects of redesigning the market to require offers to be in the form of continuous piecewise-linear functions, rather than discontinuous step functions. This change prevents degenerate solutions with non-unique dual prices in the market clearing problem, and decreases the volatility of electricity prices.

We convert publicly available historical offers into piecewise-linear format. Modifications were made to vSPD, a replica of the market clearing engine used by the market operators. We simulate market clearing using supply offers in both the step function and piecewise-linear function format. We observe that using piecewise-linear offers leads to reduced price volatility, and results in more consistent movement of price, under changing demand.

1 Introduction

This study aims to investigate the effects of redesigning the New Zealand Electricity Market so that offers are in the format of a continuous piecewise-linear (PWL) function, rather than the current discontinuous step function. Enforcing PWL offer stacks can avoid degeneracy in the market clearing problem (Discussed in Section 3.1). We also expect PWL offer bids to result in less volatile electricity prices, and steadier changes to price under perturbations in demand.

We aim to test our predictions by simulating market clearing using PWL supply offers. Past studies have been conducted to investigate the effect of supply offers by examining the effects at a single node, or a small network of nodes.[2]. There have also been studies [5, 6] using piecewise-linear supply curves in economic analysis of electricity markets. As far as we are aware, there have not been any previous studies that simulate market clearing using continuous PWL supply offers, on a whole national grid network.

By modifying the publicly available vSPD program, released by the Electricity Authorities, we simulate market clearing in the New Zealand wholesale electricity market using PWL offers. We empirically show that price volatility decreases under the PWL offer scheme, and sudden jumps and drops in prices have largely been smoothened out.

2 New Zealand Electricity Market

The New Zealand Electricity Market is made up of generators, retailers, distributors, and the national grid. Electricity is traded at the wholesale level on a spot market. Operations on the electricity wholesale market are overseen by the market regulator, the Electricity Authority. Trading occurs at over 280 nodes across New Zealand's national grid, of which around 60 are grid injection points, and over 220 are grid exit points.

Transpower also acts as the system operator. Supply offers are submitted by generators, while demand bids are submitted by buyers. The system operator then clears the market at least cost and determines the electricity prices at each node. Market clearing is done in half hour trading periods and uses the Scheduling, Pricing and Dispatch engine.

2.1 Supply Offers

Supply offers from each generator can contain up to 5 price bands. Prices and the quantities of electricity offered at each price, are referred to as tranches. Generators can change their offer for each half-hour trading period. Generators cannot offer a quantity which exceeds reasonable estimates of the quantity of electricity the generator can supply. All submitted prices must be non-negative, but there is no limit on the maximum price. Each tranche much have a higher price than the preceding tranche, if a preceding tranche exists. A sequence of up to 5 offer tranches together is known as an offer stack.

2.2 The Market Clearing Problem

At each half-hour trading period, the market clearing problem is solved. The objective of this problem is to meet the electricity demands at minimum cost. The laws of physics apply to the transmission of electricity. In the transmission network, the flow of electricity on any line must be below the line capacity of that line. Due to Kirchoff's laws, across any loop in the network, the sum of flows multiplied by the impedance of the corresponding lines is equal to zero.

The market clearing problem can be expressed as a minimisation problem. We will use the following notation:

- T is the index set of all offered tranches;
- A is the node-arc incidence matrix;

- x is the vector of quantities dispatched, indexed by $t \in T$. x_t denotes the actual quantity of tranche t dispatched;
- q is the vector of offer quantities, indexed by $t \in T$;
- p is the vector of offer prices, indexed by $t \in T$;
- f is the vector of flows on the lines, indexed by $l \in A$;
- K is the line capacity;
- M is a $N \times T$ matrix, where each column of M corresponds to a tranche t. $M(n,t) = 1$ at where tranche t is offered at node n. All other elements in M are set to 0;
- B is a matrix obtained from A by replacing the non-zero entries by $\frac{1}{2}\rho_l$, where ρ_l is the loss coefficient of line $l \in A$;
- L is the loop constraint matrix. Each row of L represents an arc, and the columns correspond to arcs.

We can then write the market clearing problem as:

$$\begin{aligned} \text{[MCP] } & \min p^T x && (1) \\ \text{s/t } Mx + Af - Bf^2 &= d && (2) \\ Lf &= 0 && (3) \\ -K \leq & f \leq K && (4) \\ 0 \leq & x \leq q && (5) \end{aligned}$$

- The objective 1 being minimised is the total cost of generation;
- Constraint 2 ensures that flows in the network are conserved;
- Constraint 3 stems from Kirchoff's laws;
- Constraint 4 ensures that flows are within line capacities;
- Constraint 5 enforces the quantity dispatched at each tranche to be less than quantity offered at the corresponding tranche, and be non-negative.

Note that the first constraint has a non-linear term $-Bf^2$. The Scheduling, Pricing and Dispatch engine approximates the non-linear feasible region with linear pieces. The resulting problem then becomes linear.

3 Motivations for Piecewise-Linear Supply Offers

3.1 Avoid Degenerate Solutions of the Market Clearing Problem

Degenerate solutions may occur when solving the market clearing problem, if the supply offers are in the format of stepwise offer stacks. Degenerate solutions occur when the market clearing quantity lies exactly on the boundary of two offer stacks. When this occurs the market clearing price, given by the dual of the MCP, is non-unique. An example of a degenerate solution in the MCP is shown in Figure 1. Hogan[4] discusses the issue of multiple-prices in the electricity market, and proposes adding a small quantity ϵ when degeneracy occurs. The method of adding ϵ is used by the California Independent System Operator(CAISO)[7]. Under the PWL supply scheme, degeneracy of the MCP will not occur, as the supply function becomes continuous, and at any quantity on the supply function, there exists a unique price.

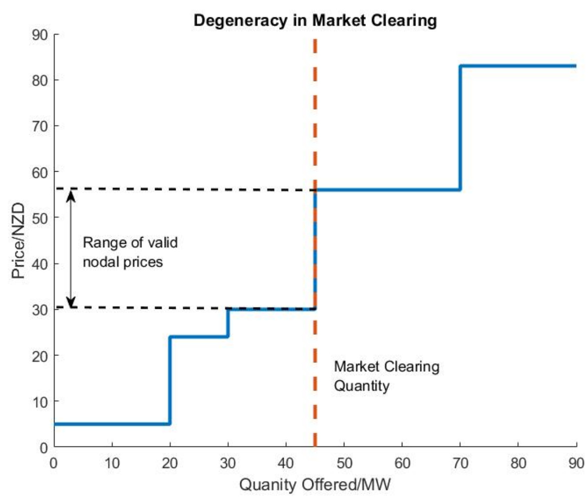


Figure 1: Degeneracy of the Market Clearing Problem

3.2 Smooth and Less Volatile Prices Under Demand Fluctuations

Deregulated wholesale electricity markets exhibit levels of price volatility unparalleled in traditional commodity markets. This volatility is often attributed to the inelastic nature of the demand, and perishable nature of electricity.[3]

Particular focus lies in examining the movement of price, under small fluctuations in demand with the current stepwise supply offers, compared to the proposed continuous piecewise linear supply offers. We wish to smoothen out jumps and drops in price when the change in quantity demanded is small, resulting in overall less volatile electricity prices. Hence ideally, small increases in quantity demanded should be met with relatively consistent small increases in price.

Industrial consumers will often have a threshold for electricity price, high prices over the threshold can force large industrial users to terminate operations. Under the stepwise offer scheme, consumers may find it difficult to anticipate how much more demand is required before the threshold is reached. A consumer may believe that the threshold is very far from being reached, but an incremental increase in the quantity demanded may result in a sudden jump to a price significantly over the threshold. When the jumps and drops have been smoothened out, each incremental increase in quantity demanded is guaranteed to result in an increase in price. Industrial consumers will be able to better adjust electricity usage according to the price signals.

4 Construction of Piecewise-Linear Supply Offers

If the format of offers in the market were to be redesigned, suppliers will be able to freely submit their offers in the designated continuous PWL format. Therefore, given historical offers, predicting the exact supply offer a generator would submit under the piecewise-linear supply scheme would be difficult. Our simulations are based on PWL offers converted from historical stepwise offer stacks. It is difficult to identify the exact strategies each generator would follow, if the supply format is changed. There are many ways to construct PWL offers from stepwise stacks, we make the following assumptions:

- Most tranches in the current offer stacks closely follow the marginal costs of generators;
- Generators will want to construct their piecewise-linear supply offers in a way such that, given the same quantity dispatch amount, the nodal prices under the two supply formats do not differ by a significant amount;
- The total revenue over all the generators do not differ by a large amount under the two supply formats.

For experiments conducted in this report, we construct the piecewise-linear offers as follows:

- k is the total number of tranches in the offer stack, and $k \leq 5$;
- i is the index of the tranches in the offer stack;

$$i = 1, 2, ..., k \tag{6}$$

- p_i denotes the price of tranche i;

$$p_i = p_1, p_2, ..., p_k \tag{7}$$

- q_i denotes the quantity offered in tranche i;

$$q_i = q_1, q_2, ..., q_k \tag{8}$$

- p'_j denotes the prices at the ends of each piecewise-linear pieces, where $j = 1, 2, ..., k+1$;

$$p'_j = \begin{cases} p_j \text{ if } j = 1 \\ p_k \text{ if } j = k+1 \\ \frac{1}{2}(p_j + p_{j+1}) \text{ if } 1 < j < k+1 \end{cases} \tag{9}$$

- g_i denotes the gradients of each piecewise-linear piece.

$$g_i = \frac{p'_{i+1} - p'_i}{q_i} \tag{10}$$

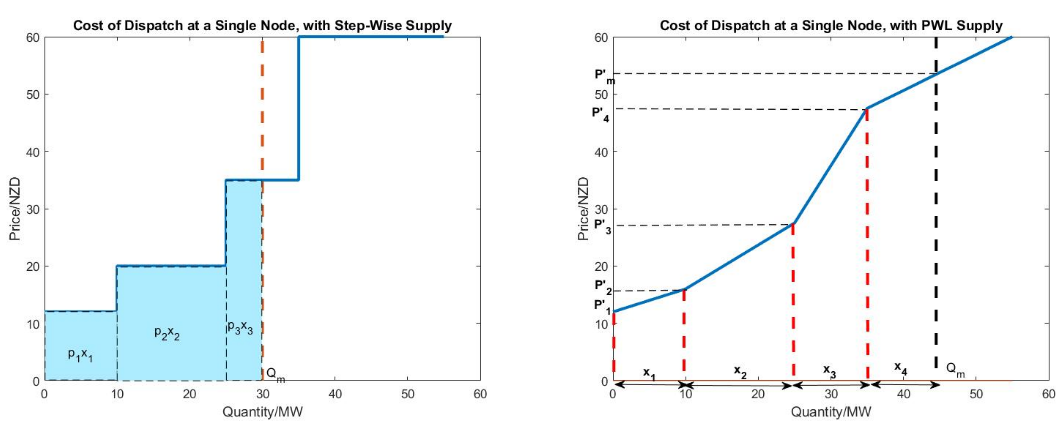
The offer price at any dispatched quantity can then be found by linearly interpolating between neighbouring p'_i values. If we denote T as the total quantity of electricity dispatched at the supply node, and $P(T)$ as the price at quantity T, then:

$$P(x) = g_i(T - \sum_{m=1}^{i} q_m) + p'_i \tag{11}$$

$$\text{where, } i \text{ is the solution to } \max_{\alpha}\{\sum_{m=1}^{\alpha} q_m \leq T\} \tag{12}$$

5 Modification of Market Clearing Problem

The objective of the market clearing problem is to minimise the total cost of dispatching the required electricity. This total cost is represented by the area under the aggregate supply curve, up to the total quantity dispatched. The total cost can be viewed as the sum of all the areas under individual supply functions, up to the quantity dispatched at each corresponding node, over all nodes. In the stepwise supply scheme, the cost at a specific node can be calculated by summing up rectangular areas under supply stacks, up to the dispatch quantity. This is represented in the Objective 1, and illustrated in Figure 2a.



(a) Market clearing with stepwise supply offers

(b) Market clearing with piecewise-linear supply offers

Under the PWL supply scheme, the cost of dispatch under a single generator can be calculated by summing up the areas of trapeziums under the PWL supply function. This is shown in Figure 2b, and can be expressed as the following:

- p'_i is the lowest price of each linear piece i of the supply function. Note that p'_i is not equal to the price (p_i) of the corresponding stepwise offer tranche
- p'_m is the dispatch price at market clearing quantity Q_m
- q_i is the maximum quantity offered at linear piece i
- x_i is the quantity of electricity dispatched for linear piece i of the supply function, $q_i \geq x_i$. Due to market clearing being an optimisation problem, if $x_j \neq 0$, for some $j \neq 1$, then $x_1 = q_1, ..., x_{j-1} = q_{j-1}$, i.e. For a given node, before we dispatch at a new linear piece, all lower-priced linear pieces must be fully dispatched

$$\begin{aligned} DispatchCost &= \sum_{i}^{n} A_{trapezium_i} \qquad (13)\\ &= \sum_{i}^{n-1} 0.5x_i(p'_i + p'_{i+1}) + 0.5x_n(p'_n + p'_m) \qquad (14)\\ &= \sum_{i}^{n} 0.5x_i(p'_i + p'_i + g_i x_i) \qquad (15)\\ &= \sum_{i}^{n} 0.5x_i(2p'_i + g_i x_i) \qquad (16)\\ \text{Where,} x_1 &= q_1, x_2 = q_2, ..., x_n \leq q_n \qquad (17) \end{aligned}$$

The market clearing problem(MCP) for the wholesale market in the vSPD engine will need to be modified. The objective of the MCP is to minimise the total cost of generating the quantity of electricity to satisfy demand, this is equal to the area under the price curve up to the quantity dispatched at the given node. After deriving p'_j and g_i, and the dispatch cost (Equation 13), the modified MCP can be expressed as:

$$\begin{aligned} \text{[Modified MCP]} \ \min \sum_{i} 0.5(2p'_i + g_i x_i)x_i \qquad (18)\\ \text{s/t } Mx + Af - Bf^2 = d \qquad (19)\\ Lf = 0 \qquad (20)\\ -K \leq f \leq K \qquad (21)\\ 0 \leq x \leq q \qquad (22) \end{aligned}$$

As shown above, the modified MCP contains a quadratic objective function, rather than a linear objective function. The objective of the modified MCP is convex, and continuous. The continuity of the objective results in a unique price for any valid market clearing quantity.

6 Experiments

Experiments were conducted to simulate market clearing under the two different supply offer schemes. Our main aims to achieve through our experiments include:

- Analysing the volatility electricity prices at different nodes under realistic oscillations of demand
- Investigate whether the switching to a PWL supply scheme results in steady, and more predictable increases in prices when electricity consumption increases.

Our experiments simulate market clearing in the New Zealand wholesale electricity market, utilising the publicly available supply offers and demand bids, and the vSPD program.

We assume that the piecewise linear supply offers put in by suppliers will be similar to those we have obtained through conversion from stepwise offer stacks. This assumption will hold if both the stepwise offer stacks and PWL offers are accurate reflections of the marginal costs of generation for the supplier.

6.1 Simulating Fluctuating Demand

In our experiments, we want to simulate fluctuating demand, while supply offers remain unchanged. We assume small perturbations in demand can be mimicked by multiplying the demand bids at the grid exit points by scale factors sampled from a log-normal distribution, with $\mu = 1$ and $\sigma = 0.025$. Two correlated scale factors (F_{North} and F_{South}) are extracted for the North and South Island nodes. The correlation factor used is $r = 0.4$. These scale factors are generated as shown:

$$Z_{North}, Z_{South} \sim N(0, 1) \tag{23}$$

$$F_{North} = \exp\left(\mu + \sigma Z_{North}\right) \tag{24}$$

$$F_{South} = \exp\left(\mu + \sigma^2 Z_{North} + \sqrt{1 - r^2} Z_{South}\right) \tag{25}$$

7 Experimental Results

7.1 Smoother Prices with Fewer Jumps

We have simulated market clearing under fluctuating demand (method of generating fluctuations outlined in 6.1) at different trade periods of the day, and on different days in the year 2016. The prices for Benmore power station at specific trading periods are shown in Figure 3.

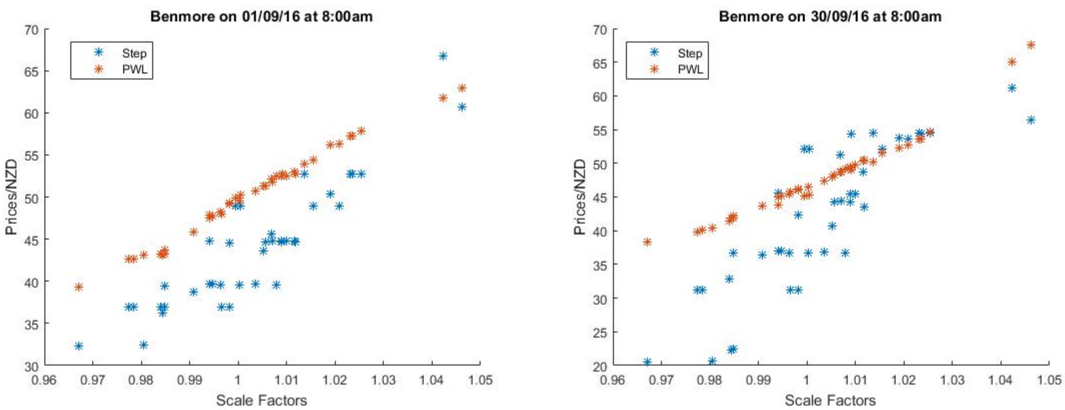


Figure 3: Prices under fluctuating demand at Benmore Power Station at different trade periods. Y-axis denotes the price, while x-axis denotes a weighted average of the scale factors of both islands, weighted on the number of generators on each island.

When demand is perturbed, the price values resulting from PWL offers appear to have a much more consistent pattern, while the price values resulting from the stepwise offer stacks are much more scattered. Under the stepwise supply scheme, sudden jumps in prices exist. Under the PWL supply scheme, the price of sample points roughly follow a continuous monotonously increasing trend with respect to increases in scaling factors. The prices under the two supply schemes are relatively similar when the scale factor is approximately equal to 1.

7.2 Reduced Volatility

The reduction in volatility is also evident under the PWL scheme. The mean and standard deviation figures, over 40 fluctuations, for our experiments are tabulated and shown in Table 1. We can see that the mean prices are similar under both schemes, but the standard deviation is almost always lower under the PWL supply scheme, indicating a lower level of volatility.

Table 1: Prices at Benmore using stepwise and PWL supply schemes, at 3 different times, over 4 different days in September. The smaller standard deviation is in bold.

		1st		10th		20th		30th	
		Step	PWL	Step	PWL	Step	PWL	Step	PWL
8:00am	Mean	44.2	50.4	52.8	51.1	66.7	67.5	42.2	47.3
	S.D.	7.4	**5.4**	6.9	**6.1**	22.4	**17.6**	10.8	**5.9**
1:00pm	Mean	54.3	53.5	32.6	34.3	52.4	56.3	53.6	47.4
	S.D.	5	**3.7**	8.6	**3.5**	6.6	**6.1**	**4.7**	7.1
8:00pm	Mean	56.1	60.2	43.2	43	56.7	62	53.5	60.3
	S.D.	9.7	**3.5**	6.8	**4.8**	13.1	**7.9**	9.7	**7.5**

8 Conclusions

An empirical study was conducted to investigate the effects of redesigning the New Zealand Electricity Market so that piecewise-linear supply offers, instead of the current stepwise tranches, are to be submitted by suppliers. Market clearing under the piecewise-linear supply scheme will prevent degeneracy of the market clearing problem, and is expected to decrease the volatility in the market.

Through modifications of the vSPD program, we were able to simulate of market clearing with piecewise-linear supply offers. Using publicly available historical data on offers and bids, we simulated market clearing on trade periods in 2016, using both stepwise offer stacks and piecewise-linear offers. It was found that the electricity prices resulting from piecewise-linear supply offers are much less volatile, exhibit fewer jumps, and monotonically increase as demand is scaled up.

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