

# Three Dimensional Container Packing of Drums and Pallets

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## Abstract

A manufacturer of electrical cable needs to pack drums and pallets of cable into standard shipping containers for exporting overseas. Packing is done in such a way as to minimise the number of containers used. A three dimensional packing heuristic is developed to solve this problem. The packing plans prepared manually were compared to the plans obtained from the heuristic. The heuristic was found to increase the average utilisation of the containers. In some cases the heuristic eliminated freight which had been sent as loose cargo by fitting the entire loose cargo into the current number of containers. An analysis of the drum sizes used in the problems found that some sized drums caused large inefficiencies. Some of the drums required small reductions in their diameter to allow for even higher container space utilisation.

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## 1. Introduction

BICC Cables New Zealand Limited is a manufacturer of copper electrical cable destined for both the New Zealand market and various export markets around the Pacific Rim and the Middle East. After manufacture, BICC's cable products are wound onto wooden drums which are then packed into standard shipping containers. Packing staff manually determine an appropriate way to fit the drums into the container(s) using their packing experience and common sense. In some instances the number of full containers cannot accommodate the entire order. Unpacked drums are then sent in either an additional container or as loose freight in a loose container lot (LCL) which are subject to extra costs and delays. The problem is to develop a packing plan for the drums in order to minimise the number of containers and LCL freight. In order to do this the following issues needed to be considered.

1. A drum needs to be modelled as cylindrical mass.
2. Drums must not be stacked flange down as this can occasionally result in damage to certain types of cable and would cause problems with handling.
3. Individual drums must not be placed on top of those that are more than 20% lighter. This reduces the possibility of damage to drums. The weight of a drum will be modelled as a function of its volume.

4. To avoid drums rolling around in transit, it is preferable to stack drums with the flange facing the door of the container. This also ensures a forklift can be used during loading and unloading.
5. Pallets must be incorporated into the packing, where each pallet is modelled as a rectangular cubic mass. Pallets can be stacked to a maximum of three layers high. Pallets are light enough so that they may be stacked on top of any sized drum.
6. The weight is to be distributed as evenly as possible throughout the container. A weight imbalance is potentially hazardous during shipping.

## **2. Literature on Packing Heuristics**

The problem of packing items into containers in an efficient manner can be computationally complex. Existing approaches to container loading problems usually apply to a specific class of problem encountered in practice but there are many scenarios for which no adequate methodologies currently exist. Most of the literature on packing examines computer-based solution techniques using packing heuristics applied to the packing of rectangular boxes into standard shipping containers as well as other non-standard containers. There are three major subsets of the overall study of three-dimensional box packing; single box type problems, multiple box type problems and three-dimensional pallet loading. Dowland [4] and George [7] have investigated the packing of single box types. The packing of containers with multiple box types has received wider attention, for example [9][14][6] and [12]. In addition, there are the problems of three-dimensional pallet loading. These are similar in structure to three-dimensional container loading problems and hence aspects of these approaches are applicable to the container loading scenarios. Some relevant papers are [16][15][2][1] and [13]. It has been suggested that this concentration on box packing problems is a reflection of the wide range of applications in industry for this set of problems. This focus on box-packing is also indicative of the fact that these problems are less complex than those associated with the packing of non-rectangular and irregular shapes, circular objects included.

Several papers, [3][5][10][8], have investigated the problem of packing pipes, reels and cylinders into containers. These problems exhibit some similarities to the drum packing problem. However, in all cases these problems reduce to the two-dimensional case of fitting circles into a square or rectangle. Although not able to solve the three-dimensional case the concepts incorporated in these heuristics could be used in solving various aspects of the drum packing problem.

## **3. Packing Concepts**

Packing heuristics generally use rules for placing items - such as the cable drums - into containers or onto pallets. These rules originate from observations of packing procedures and the recognition that particular configurations are more effective than others. Procedures are then developed that extend these approaches to encompass more extensive and complex combinations of items. The expectation is that a generalised packing rule that works well for a small sample of items will extend well to the efficient packing of many items of approximately the same type and size in the same proportions. Different packing rules may be applied to combinations of items of different size or shape.

An example of a well-known General Packing Rule is the “First-Fit-Decreasing” approach. Many of the packing procedures employ this rule and variations thereof [9][6][10]. For this rule, given a selection of items of different sizes, the packing rule dictates that the most awkward items are packed first. In this instance, awkward items could be defined in terms of unusual dimensions such as diameter, width, volume, weight or other aspects such as shape or fragility. The result is a ranking procedure that orders the items from the most to the least awkward. A packing procedure then selects an item to be placed in a packing pattern based upon the item’s rank. A justification for this approach is that it is easier - and more efficient in terms of minimising wasted space - to arrange small items around a large item than it is to accommodate a large item once small items have been placed.

Another approach dictates that items of the same – or similar – size or shape are placed together in the same area of a container [9]. It is recognised that some very efficient packing patterns can be achieved using this rationale. It also means that for individual item types the associated efficient packing patterns can be stored so that when enough of these items are to be packed the best packing pattern is immediately assigned. As a consequence, computer processing time can be devoted to patterns involving mixes of different items. A similar approach was suggested by Haessler [11] when developing a heuristic to minimise trim loss in a paper roll cutting application.

Once the ranking procedures have been used to select the next item to be placed, the item is packed in a layer in such a way as to maximise the volume utilisation of the usable space. Often this is achieved by employing a procedure that will allow for the removal and rearrangement of a number of the most recently packed items. This can be done numerous times, using various combinations of orientations until the most efficient configuration is obtained. The algorithm will then select the best configuration at that point and move on to the next item or set of items.

Software developed for packing is often implemented by packing each item using one - or usually more - of the packing heuristics. Items are placed into each container until the list is empty or until the container(s) is (are) full.

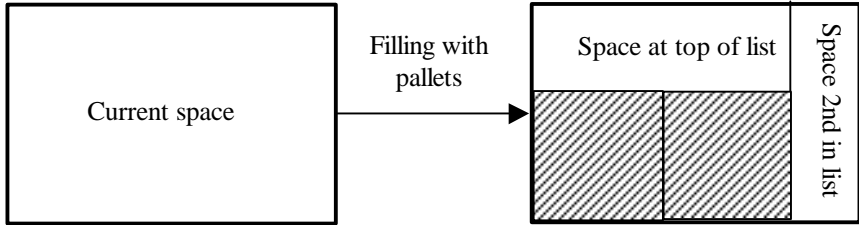
#### **4. The Packing Heuristic**

The heuristic developed is based upon the concept of filling each container using a cross-sectional layer-by-layer approach. Initially pallets are packed into the container using a simple heuristic based on the ideas from George and Robinson [9]. This is done first because of the awkwardness of the pallets. A description of the heuristic used is outlined in Figure 1.

Each drum is manually defined as either being a drum that can be used to create an entire layer or a drum that is used to make up a layer with other drums. Layers that can be made up of a single drum type or a composition of the same diameter drums are packed first. Once these “complete” layers have been packed the general packing heuristics begin.

Each layer is broken down into rows. Four different methods of packing the row are tested, and the method that packs the most volume is chosen. Ties are split by choosing the solution that produces the highest density of drums. A new layer is started when the previous layer cannot accommodate any more drums. Layers are packed until the end of the container is reached. Once this happens, a new container is commenced. The

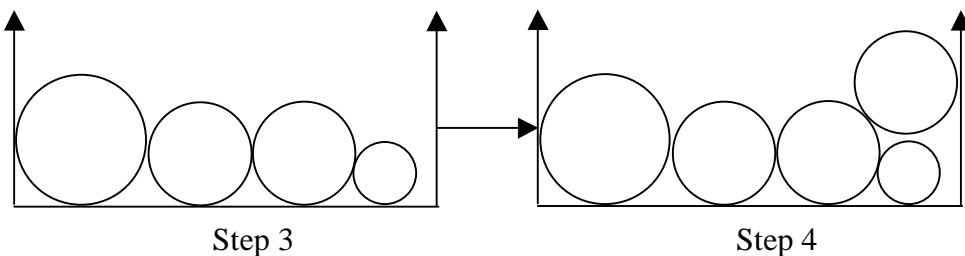
heuristic is complete when no more drums remain. A summary of the heuristic is outlined in Figure 2a and b.

1. Add empty container size to list of available spaces
  2. Select the space at the top of the space list
  3. Select the highest ranking pallet type (by volume) which can fit into the current space. If none then go to 8
  4. Pack the pallet type widthways or lengthways depending on what leaves the smallest remaining space.
  5. Update pallet quantities by subtracting the number of pallets packed from the number of pallets remaining to be packed.
  6. From the current space, the area to the right of the pallets is added to the top of the space list
  7. From the current space, the area above the pallets is added to the top of the space list
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The diagram illustrates the process of filling a 'Current space' with pallets. An arrow labeled 'Filling with pallets' points from a large empty rectangle on the left to a smaller rectangle on the right. The right rectangle is divided into a shaded area at the bottom representing the packed pallets, and an unshaded area above it. The unshaded area is further divided into two vertical sections: the top section is labeled 'Space at top of list' and the bottom section is labeled 'Space 2nd in list'.
8. Remove the current space from the space list
  9. If there are no pallets left then go to 11
  10. If there are spaces left then go to 2
  11. End Pallet Packing

Figure 1. Pallet Packing Algorithm

1. Rank drums according to volume
2. Pack drums which have been defined as being able to be packed in one layer and have sufficient volume to fill an entire layer, with the same drum type or with a drum type with the same diameter.
3. Start a new layer by packing the bottom row from left to right choosing drums in order of the highest rank drum that will fit the space available. End the layer when no more drums will fit.
4. Add the highest ranking unpacked drum to the right hand side wall.



5. Pack the next of the row by selecting the best solution which packs the highest volume of drums from the following methods. Ties are resolved by choosing the solution with the smallest absolute deviations between the drums centre points, hence providing a denser pack.

Figure 2a. Drum Packing

- 5.1 Pack from right to left the highest ranked drum that will fit into the “valleys”, i.e. the recess which results from two adjacent drums or a drum and a wall, of the drums in the previous row. If the valley is big enough and the current drum small enough, more than one drum may be inserted into the valley. End the row by packing the drum as low as possible against the left-hand wall.
- 5.2 Pack from left to right the highest ranked drum that will fit into the “valleys” of the drums in the previous row. Again more than one drum may be inserted into the valley. End the row by packing the drum as low as possible against the left-hand wall.
- 5.3 Pack the highest ranked drum that will fit as low as possible against the left-hand wall, then pack from right to left the highest ranked drum that will fit into the “valleys” of the drums in the previous row. Again more than one drum may be inserted into the valley.
- 5.4 Pack the highest ranked drum that will fit as low as possible against the left-hand wall, then pack from left to right the highest ranked drum that will fit into the “valleys” of the drums in the previous row. Again more than one drum may be inserted into the valley.
6. Pack the next row in the same manner as step5, except insert the single drum on the right-hand wall rather than the left-hand wall.
7. Repeat steps 5 and 6 until no further drums can be inserted in the layer
8. Allow drums to be rolled into the container in the last layer. Drums can also be packed without a supporting drum underneath. Ie. ‘cram’ drums in anyway possible into the last layer and create braces to hold them in place during transit.
9. If a drum is of a size such that only one drum can fit into the cross-section of the container, then this drum may be rolled in. Since there are few drums of this size, every combination of rolling-in and flange-outward packing is tested and the best selected. Eg. If the order contains three ‘large’ drums, 8 ( $2^3$ ) different solutions are obtained and the best chosen.

Figure 2b. Drum Packing

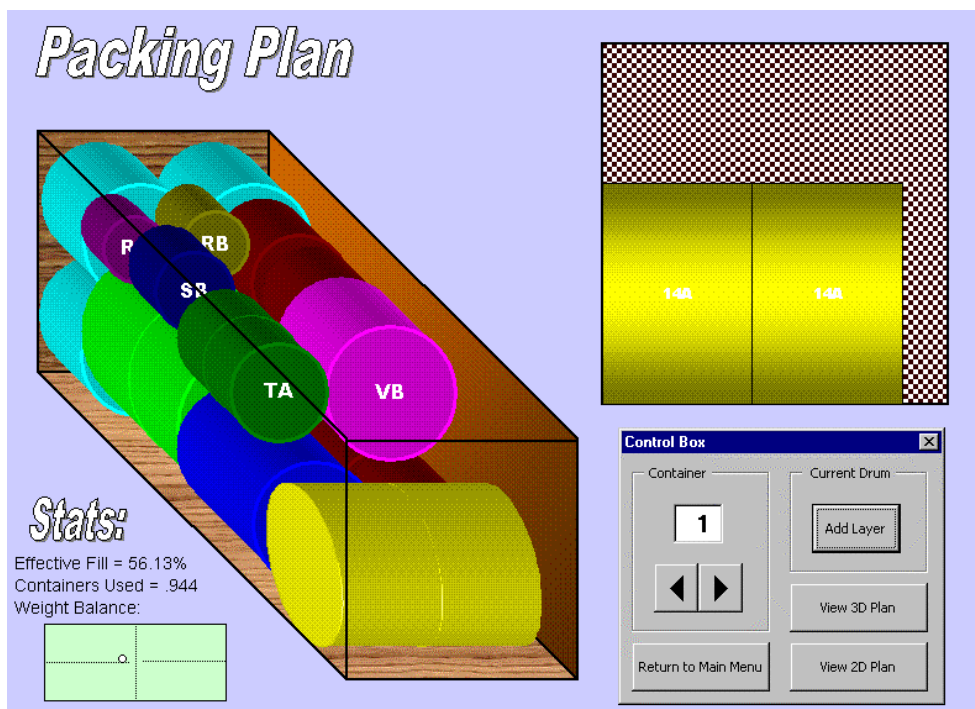


Figure 3. A Sample Packing Plan

## **5. Software Implementation**

The company required a package that was very user friendly, reliable and able to produce good packing results. The prototype for the system, called WOLFPACK, was coded in Visual Basic for Applications and Microsoft Excel 97.

The WOLFPACK software allows the user to define drum sizes, enter, load and save orders and then ultimately produce a packing plan for the orders. This packing plan can be in the form of a two-dimensional packing plan or a three-dimensional packing plan which is progressively revealed layer by layer. A sample packing plan is shown in Figure 3. The packing plans also provide performance details of the plan in terms of the container fill-rate and where the weight balance is centred for the container.

## **6. Data Sets**

In order to assess the performance of the heuristic it was tested on export orders of a size requiring more than one container load; that is, one container plus additional LCL or two or more containers. The expectation was that either the number of containers used might be reduced, or the total volume of any resulting LCL might be decreased.

The criterion for selection of testable orders of multiple container loads was that the first containers loaded had to be fully packed so that items in the last container packed could not conceivably be packed into the first containers.

Twenty-one orders that met the selection criteria were used in the computational tests.

For the purposes of this evaluation, when considering two or more container loads for which there was no actual LCL, total drum volume in the final container loaded was treated as notional LCL. The measure of solver performance was the reduction in this actual and notional LCL volume. The decision as to whether the remaining volume be sent as LCL or in an additional container is left to the company based on relevant cost information.

## **7. Drum Sizes**

The following section discusses a simple yet very effective approach to packing problems that can easily be overlooked in favour of rather more technically and mathematically rigorous approaches. Faced with a complex combinatorial problem such as packing a manufacturer's product, and a set of given product and container sizes, a complex heuristic can be applied to find an acceptably good solution. One can also "change the givens". Combining the heuristic with a set of altered product and/or container sizes can provide very good solutions.

Due to the nature of orders from overseas customers, significant proportions of export orders consist of large numbers of only a few drum types (in a sample of 20 export orders, 13 drum types made up approximately 85% of the total number of drums). As a result, some containers will have the following characteristics; several layers of drum type A, followed by some layers of drum type B, followed by a few layers of drum type C, ending with a mixture of the remaining drums types in the final few layers.

For any particular layer consisting of a single drum type, the packing staff did tend to use the most efficient packing pattern possible. However, observations of loading procedures at the company highlighted inefficient layer utilisation when some of the

single drum types were packed. Given that drums of one type are packed in a layer, the current range of drum types have volume utilisation between 45.5% and 74.6% with an average over all types of 64.4%.

These inefficient single layer utilisation can be attributed to two separate dimensional constraints; those of the shipping container, and those of the drums themselves. The shipping container dimensions are fixed absolutely. Hence, investigations focussed on altering the dimensions of the drums. This could be done by altering the external drum dimensions – the flange diameter and/or external traverse (i.e. drum depth) see Figure 4.

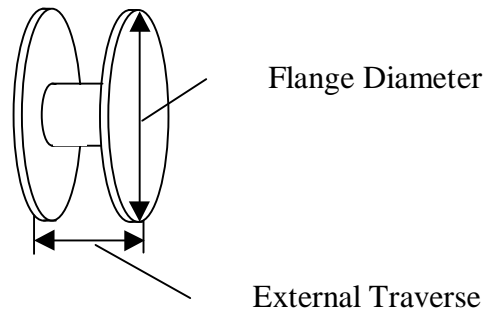


Figure 4

Faced with the observations discussed above, the study initially explored appropriate drum dimensions for packing single drum types in a layer. Underlying the investigation of new drum sizes were several important considerations (aims):

1. The patterns should – where possible – improve upon current layer utilisation.
2. The layers should be easy to load and unload.
3. The layer patterns should preferably be simple and easy to replicate by packing staff.
4. The layer configurations should be stable.

There were also certain constraints that restricted the choice of new sizes:

1. Dramatic changes to drum sizes were seen as undesirable by the company. Very long traverse was the least favoured option because of the impact on handling and difficulties a long traverse might impose upon the mechanisms for winding cables onto the drums. Also, a small traverse in relation to the flange might make drums in loose storage more prone to toppling over. The authors decided that the external traverse should not be less than 35% of the flange diameter.
2. Cost of flange construction represents a significant proportion of the drum manufacturing cost. Therefore, any increase in flange diameters should be kept to a minimum.
3. Because the dimensions of the shipping container are fixed, flange diameters are limited to an upper-bound of 2.33m (the minimum of a container's width and height).
4. The internal useable volume for each drum type should remain unchanged from present values. Present drum type selection procedures (given an order of a particular cable type and length) were to remain unchanged, and internal useable volume is one of the determining factors in this selection procedure.

The width and height dimensions of a standard 20ft or 40ft shipping container are 2.33m and 2.39m respectively. Given that this drum-packing layer boundary is

essentially square in shape, simple grid-type patterns are known to produce very good volume utilisation for packing circles in squares [10] whilst also satisfying the aims of simplicity and load stability.

Since the container width was the constraining dimension, the general principle was to alter the flange diameters in order to fit an integer number of drums evenly across the width of a container. A small allowance was made for the fact that there was some variability in flange diameters compared with the drum manufacturer’s specifications. Following restrictions 1 and 2 above, in the first instance, drum dimensions were altered as little as possible from present dimensions, whilst still achieving improved container space utilisation. For example, one particular drum type has a current flange diameter of 700mm resulting in 9 drums to a layer and a utilisation of 66.2%. The flange can be decreased to 575mm or increased to 775mm. The increase is less than the decrease, therefore the diameter is set to 775mm. Nine drums still fit in a layer however, utilisation is increased to 76.6%. New external traverse dimensions were calculated based on maintaining constant internal volume.

A further aim was to ensure that drum types sharing common flange diameters would continue to share common – albeit altered – flange diameters.

Following this rationale resulted in many of the drum types sharing the same diameter. This in turn reduced the number of different flange diameters from 14 to 4. Most importantly, volume utilisation improved for all drum types, in some cases markedly. The resultant volume utilisation for single drum types in a layer was between 73.5% and 77.8% with an average of 76.6%.

For actual export orders, an evaluation of the improvement in overall container volume utilisation achieved through changes to drum size was determined using a sample of twenty single container export orders. For both sets of drum size the containers were filled using current manual packing methods. WOLFPACK was not used. The results are shown in Table 1.

	Original Drum Sizes	Resized Drums	Change
Utilisation	58.14%	65.06%	6.92%
Average Length (metres)	5.49	4.86	0.63
Times obtained Minimum Length	2	18	

Table 1. Results using Manual Packing Methods on a Twenty order Sample

There were two further benefits besides those obtained in terms of improved volume utilisation. Firstly, a significant factor in drum manufacturing costs is attributed to flange construction and in set-up times for manufacturing the different flanges. By reducing the number of flange sizes for the most frequently used drums from 14 to 4, the number of set-ups can be reduced. Secondly, some of the current single drum layer configurations - particularly for the larger and heavier drums - result in a significant component of the weight of the drums being directed against the side walls of the containers. This places stress on the containers and on some occasions, container walls have bowed so much that the containers have been repacked. The resized single drum layer configurations result in most of the drum weight acting directly downward to the floor of the containers.



## 8. Computational Results

The results of the heuristic performance evaluation are summarised in Table 2. These results apply to the twenty-one orders that were part of the evaluation sample. The Total Volume required row is the total volume used for packing the 21 orders. The Full Containers + Remaining  $m^3$  row indicates how many full containers were used and the number of cubic metres left over in the last container. This volume in the last container could be sent either as a full container or as LCL. The row Estimated Containers + Estimated  $m^3$  LCL used provides an estimate for what would be shipped assuming that if a container is approximately half full then it goes as a full container otherwise it goes as LCL. In reality the amount varies depending on where it is being shipped and the costs involved.

	Manual	Algorithm using Current Drum Sizes	Algorithm using Re-sized Drums
Total Volume Required ( $m^3$ )	1027.1	937.4	896.9
Full Containers + Remaining $m^3$	26 + 172.9	24 + 148.9	24 + 108.4
Estimated Containers + Estimated $m^3$ LCL used	35 + 40.9	31 + 52.4	29 + 41.8
Improvements on Manual Volume Required			
Average		5.35%	9.51%
Standard Deviation		6.7%	6.62%
Maximum		26.23%	29.43%
Minimum		-2.64%	0.00%

Table 2. WOLFPACK Heuristic Performance

From Table 2, it can be seen that overall, WOLFPACK out-performed current BICC packing methods using either the current drum sizes or the re-sized drums. When using the current drum sizes there were three instances where the manual method outperformed the heuristic, but on average the heuristic provided significant improvements, some of which were very large. Re-sized drums and the use of the heuristic improved on the volumes required in all but one case where the volume used was unchanged.

Using current drum sizes the total shipping volume required is reduced significantly, but more importantly the number of containers and LCL volume are both reduced by using WOLFPACK. For re-sized drums, even greater volume reductions are obtained.

## 9. Conclusion

As far as the authors are aware the three-dimensional pallet and drum packing problem has not been addressed in the literature. Therefore the prototype solver developed for the pallet and drum packing problem has no “bench-mark” with which to compare its performance other than current manual methods. This heuristic provides a bench-mark for further research into this area.

## Acknowledgements

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