Optimisation of Forest Harvesting Subject to Area Restrictions on Clearfell

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Abstract

When a forest is to be harvested an area-restriction is often imposed on the maximum area of a clearfell. This generally causes intractable complications in any deterministic optimisation process. The complexity is that of a very large and complicated integer programme, a formulation that is needed since the adjacency constraints need to be site-specific. A strategy is advanced by which certain nuclear sets of blocks can be determined. Once these are found, a vastly reduced number of adjacency constraints are required. This leads to a comparatively speedy algorithm which permits deterministic optimisation of the forest harvesting algorithm. This talk will mainly discuss ways of defining and finding these nuclear sets.

1 Introduction

In forest harvesting the present net worth of a forest is optimised subject to various strategic and site-specific constraints. Refer [3]. For management purposes the trees are arranged in *blocks*. Usually all the trees in a block will be of the same croptype, and normally they will all be harvested at once, often because of practical operational considerations. As a result of this discretisation, the formulation of a harvesting application will be MIP. The most difficult of the site-specific constraints are those needed to limit the size of any area of newly cleared land.

2 Harvesting regulations and adjacency constraints

When part of forest is felled, the exposed area is very vulnerable to erosion. This situation remains a major risk until the new crop is so well established that the soil is protected from direct rain and from sheet run-off. Generally under New Zealand conditions this takes about five years. Because of some instances of exploitation by

irresponsible forestry companies, many countries now have a maximum clearfell area regulation, which specifies the maximum area of clearfell permitted. Once such a clearing has been made, none of the surrounding forest may be cut for a certain specified length of time, the greenup period, which is usually between 5 and 10 years. It has proved very difficult to encapsulate such a regulation into adjacency constraints.

Various definitions are used to determine when two blocks are adjacent. In this paper blocks are adjacent if they share a common edge.

A common assumption is that if any two blocks are adjacent, then the harvesting of one of these will force the harvesting of the second block to be delayed until the greenup period has elapsed. This assumption greatly simplifies the problem as it ignores the significance of the actual size of each block. Murray [5] defines this type of formulation as a *unit restriction model*. The present paper is not about the unit restriction model.

In practice, the size and shape of blocks in a forest may vary considerably. Thus several small adjacent blocks can often be harvested without exceeding the maximum clearfell area. The problem of identifying violations of the clearfell regulations in such a case is non-trivial. Also the associated task of devising suitable adjacency constraints which will prevent such occurrences is very difficult too. Murray [5] defines this type of formulation as an *area restriction model*. The aim of this research programme is to develop an area restriction model which can be solved by an optimisation method.

It is conceded that if many blocks have small area relative to the maximum clearfell then the problem can easily become combinatorially impossible. In many applications it is reasonable to assume that any clearfell comprising more than a given number of blocks will necessarily be infeasible. For the present paper we are assuming this number is at most 8.

3 Literature survey

Papers by Nelson and Brodie [6] and Meneghin, Kirby and Jones [2] are representative of a deterministic approach which involves large numbers of constraints each specific to actual clusters of blocks. The very large volume of such constraints along with the attendant computational complexity makes them unsuited for large applications.

Heuristic methods, such as those by Sessions [7] and Clements [1], have proved more operationally feasible, but the quality of the solution obtained can be uncertain. Murray [5] states that little progress has been made on the optimisation of area restriction models.

4 Some definitions

A *nuclear set* is a contiguous set of blocks with the following properties. The total area of the blocks in the set is less than or equal to the maximum clearfell area. If all of the surrounding blocks were added to the set, then the total area would exceed the maximum clearfell area. The *order* of a nuclear set means the number of blocks in the set. A large forest harvesting application will always contain a very large number of

nuclear sets.

The set of blocks which are each adjacent to at least one block from a given nuclear set, but not part of this set, is called the *perimeter set*. Every nuclear feasible set has an associated perimeter set.

A *nuclear constraint* is a mathematical constraint which is specific to a given nuclear set. Primarily it enforces a limitation on the harvesting of the perimeter set whenever the nuclear set is clearfelled. It will be shown that a given nuclear constraint will often enforce the relevant limitation on other nuclear sets as well as the one it specifically matches. Consequently, the number of nuclear constraints required is much less than the number of nuclear sets.

A *restricted set* is a special type of nuclear set for which there exists one block in the perimeter such that if the area of this extra block were added to the nuclear set then the combined area would exceed the maximum clearfell area. The combination of a restricted set along with exactly one such perimeter block is called a *potential violation.* The *order* of a restricted set means the number of blocks in the set. We are assuming that the order of any restricted set is at most 8. Any violation of the maximal clearfell restrictions must necessarily involve at least one potential violation. The concept of the potential violation is used to assist in determining the minimal number of nuclear constraints required.

A *feasible chain* is a contiguous set of blocks with total area less than or equal to the maximum clearfell. No restrictions are imposed on the perimeter of a feasible chain. All the sets defined in this section are all strictly geographic concepts and carry no dependence on time or harvesting decisions.

5 How to select a necessary and sufficient number of nuclear sets

In this section we produce a procedure for finding a minimal list of nuclear sets for which the associated nuclear constraints will be sufficient to eliminate every possible potential violation for restricted sets of size 8 or less. As will be explained later, this does not imply that every one of these constraints will actually be used in a given application. Rather these nuclear sets will be used to determine any infeasibility during the optimisation process. Nuclear constraints will be added by a constraint generation process only as required to remove the actual violations detected.

The selection of nuclear sets only needs to be done once. The calculations are carried out with respect to the adjacency matrix.

Step 1 : First all nuclear sets of order 1 are chosen. After this the adjacency matrix is thinned by the removal of any adjacency relationship which does not form part of a feasible chain of order 3 with adjacencies $\{(1,2),(2,3)\}$. Notice this is the same adjacency pattern required for the next phase below. This thinning process greatly simplifies the next stage of the selection. Step 1 deals with all possible potential infeasibilities involving restricted sets of diameter 2 or less. This is the justification for the thinning.

Step 2 : Next all nuclear sets of order 3 for which the adjacencies are

$$\{(1,2),\ (2,3)\}$$

only are chosen from the thinned adjacency matrix. After this the adjacency matrix is again thinned by the removal of any adjacency relationship which does not form part of a feasible chain of order 5 with adjacencies as specified below. Step 2 deals with all possible potential infeasibilities involving restricted sets of order of diameter 4 or less. This is the justification for the thinning.

Step 3 : Then choose all nuclear sets of order 5 for which the adjacencies are

$$\{(1,2),\ (2,3),\ (3,4),\ (4,5)\},\ \text{or}\ \{(1,2),\ (2,3),\ (2,4),\ (3,4),\ (4,5)\}\ .$$

The adjacency matrix is again thinned by the removal of any adjacency relationship which does not form part of a feasible chain of order 7 with adjacencies as specified below.

Step 4 : Finally choose all nuclear sets of order 7 for which the adjacencies are

$$\{(1,2),\ (2,3),\ (3,4),\ (4,5),\ (5,6),\ (6,7)\},\ \{(1,2),\ (2,3),\ (3,4),\ (3,7),\ (4,5),\ (5,6)\},$$
$$\{(1,2),\ (2,3),\ (3,4),\ (3,6)\ (3,7),\ (4,5)\},\ \text{or}\ \{(1,2),\ (2,3),\ (3,4),\ (3,6)\ (6,7),\ (4,5)\}.$$

If the adjacency matrix is represented by its graph, then the process of thinning can be illustrated graphically, as shown in figures 1 and 2.

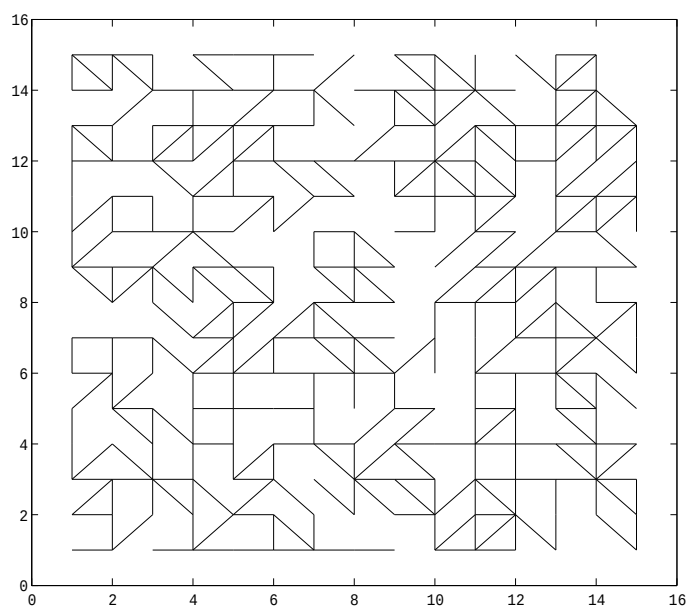

Figure 1: The graph of the initial adjacency matrix for a forest of 225 blocks with random adjacencies and random block sizes. The maximum and minimum block areas are respectively 0.8 and 0.05 as a proportion of the maximal clearfell permitted.

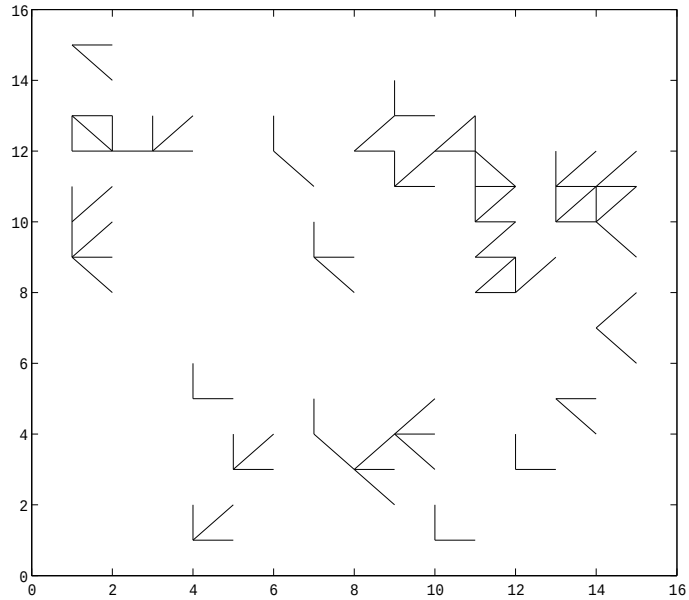

Figure 2: The same forest as Figure 1 with the adjacency graph thinned so as to retain only feasible chains of order 3 or more.

6 The small number of nuclear sets required

Table 1 shows the results of simulation trials using a forest consisting of 225 blocks with random generation of block areas and adjacencies.

adjacency probability	maximun block area	minimum block area	number of nuclear sets needed order 1	order 3	order 5	order 7	total
.5	1.0	0.1	145	9	0	0	154
.8	1.0	0.1	198	8	0	0	206
.5	0.6	0.1	64	33	0	0	97
.8	0.6	0.1	146	82	5	0	233
.5	0.4	0.1	18	74	2	0	94
.8	0.4	0.1	68	199	53	0	320
.5	1.0	0.05	135	17	0	0	152
.8	1.0	0.05	191	21	0	0	212
.5	0.6	0.05	65	66	23	4	158
.8	0.6	0.05	116	93	26	0	235
.5	0.5	0.05	29	61	19	1	110
.8	0.5	0.05	92	147	133	21	393

Table 1: Results from simulation trials with a forest of 225 blocks.

The key finding from table 1 is that the actual number of nuclear sets needing to be chosen is generally relatively small. In fact it is of the same order of size as the number of blocks in the forest. It is assumed we have a forest of 225 blocks represented by the

integer points on a square grid. For each trial an adjacency relationship occurs between neighbouring blocks i and j with probability p. The area of each block is randomly assigned with uniform distribution between a designated minimum and maximum, with both measured as a proportion of the maximum permitted clearfell.

7 Clearfell violations with nuclear sets

Once the optimisation process has begun, clearfell violations are identified by scanning the block variables associated with each chosen nuclear set in turn, timewise. If all the blocks in a set are harvested within the span of the green-up period, then the perimeter set is checked for any violation. This process is a key part of the solution algorithm. Success depends upon the fact that the number of nuclear sets involved is relatively small. Each violation found will be specific to a given nuclear set and to a certain time interval.

8 Nuclear adjacency constraints

Let S be a nuclear set, and P be the associated perimeter set. Let T be the length of the greenup period. Suppose the blocks in set S are designated for harvest between time periods t_a and t_b inclusive, with $t_b - t_a \leq T$. Let x_{it} be a binary variable with $x_{ij} = 1$ when block i is to be harvested in year t. Let a_i be the area of block i, A_S be the area of set S, A be the maximum clearfell area, and M be a suitably large constant. Then the required nuclear constraint may be written

$$M \sum_{i \in S} \sum_{t=t_a}^{t_b} a_i x_{it} + \sum_{j \in P} \sum_{t=t_b-T+1}^{t_a+T-1} a_j x_{jt} \leq A - A_S + M A_S. \tag{8.1}$$

The strategy during the solution algorithm is that only a very small number of these constraints will be included explicitly. For each violation found just one constraint will be included and the model then re-optimised. As explained above, each violation is specific to a particular nuclear set and to a specific time interval.

9 Aspects of the solution algorithm

Here is a formal summary of the algorithm.
Step 0 : First select all necessary nuclear sets.
Step 1 : Solve the relaxed LP using column generation.
Step 2 : Test by scanning all nuclear sets obtained in step 0. If adjacency violations occur go to step 3; else go to step 4.
Step 3 : Add appropriate nuclear adjacency constraints. Go to step 1.
Step 4 : If fractional values of integer variables are present then go to step 5; else go to step 6.
Step 5 : Continue the branch and bound process by adding another constraint branch.

Then go to step 1.

Step 6 : If this integer solution is within an acceptable interval then stop; else backtrack on the branch and bound tree. Return to step 1, or stop if the tree is exhausted.

10 Conclusions

This is part of an on-going research programme, being a sequel to the paper I gave at the ORSNZ conference in 2001 [4]. In that paper I also demonstrated the complete solution algorithm for an actual harvesting problem. This showed that not only was the number of nuclear sets relatively small, but also the number of the related nuclear constriants actually needed in the solution process was also small.

The practical problem being addressed is recognised as an emerging problem of very considerable operational importance. It is hoped that this contribution along with other current research initiatives will result in a commercial package being available in the not too distant future.

References

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