

# Biobjective Air Traffic Flow Management

Kenneth D. Kuhn  
Civil and Natural Resources Engineering  
University of Canterbury, Christchurch, New Zealand  
kenneth.kuhn@canterbury.ac.nz

Andrea Raith  
Engineering Science  
University of Auckland, Auckland, New Zealand  
a.raith@auckland.ac.nz

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## Abstract

Air traffic flow management seeks to extenuate delay created by congestion in the air traffic control system while ensuring equitable access to air transportation system resources. Mathematical programming formulations of the air traffic flow management problem typically minimize delay costs, ignoring evidence that equity is a critical concern in practice. Recently, authors have adjusted classical formulations, adding terms to the objective function to penalize various results deemed unfair. This work reformulates the air traffic flow management problem as a formal multiobjective optimization problem. We are able to find all Pareto-optimal solutions trading off efficiency and equity, without having to select and parameterize a model of the costs of inequity.

**Key words:** air traffic flow management, multiobjective optimization, equity, air traffic control.

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## 1 Introduction

There often arise situations where the numbers of aircraft that are scheduled to use certain airports or fly through certain sections of airspace over five to fifteen minute periods of time cannot be safely and efficiently accommodated. The numbers of aircraft that can safely land and take off at airports are largely determined by visibility, wind, and weather conditions that can change relatively rapidly. It is worth noting that in the United States air carriers often set schedules based on assumed optimal weather conditions. In certain busy sections of airspace, local demand for air traffic control services can threaten to outstrip the safe and efficient

operating capacity of the system. As past researchers have noted, this happens in western Europe “on an almost routine basis” (Lulli and Odoni 2007). Air traffic flow management (ATFM) involves strategically altering flight schedules to avoid and mitigate local system capacity deficits.

Ground delay programs (GDPs) are commonly used tools of air traffic flow management. During GDPs, flights destined for capacity constrained airports are delayed on the ground at their origin airports prior to take off. The costs, per unit time, of delaying a flight on the ground are significantly lower than the costs of delaying that same flight once airborne (Bertsimas and Patterson 1998). Airspace-flow programs (AFPs) control the rate at which aircraft arrive in capacity constrained sections of airspace. Setting appropriate GDPs and AFPs to avoid multiple potential local air traffic control demand-capacity imbalances across a network is clearly a challenging task. ATFM decision making today follows heuristic procedures, but past researchers have noted that “computer-based decision support systems might improve ATFM performance significantly” (Lulli and Odoni 2007).

There have been significant research efforts formulating and solving various mathematical programming formulations of ATFM problems. Much of the available research focuses on managing air traffic arriving at a single airport. Considering capacity constraints associated with multiple airports and sections of airspace, while allowing for en-route speed adjustments complicates the problem. The most commonly cited formulation of “the air traffic flow management problem” (Bertsimas and Patterson 1998) considers such a problem. Stochastic formulations have addressed the issue of uncertainty in airspace and airport capacities. Researchers have extended formulations to capture the dependence between the arrival and departure capacities of airports, as well as to consider the possibility of rerouting aircraft. Many researchers have investigated the computational performance of various formulations and solution strategies for ATFM problems (Bertsimas and Patterson 1998; Hoffman and Ball 2000).

The “fundamental principle” of air traffic flow management in the United States today is *Ration by Schedule (RBS)* (Gupta and Bertsimas 2010). According to RBS, aircraft are assigned slots at a capacity constrained airport according to a schedule that preserves the order with which the aircraft were originally scheduled to land. The Collaborative Decision-Making (CDM) paradigm, also in widespread use today, holds that ATFM decisions are made with significant authority and responsibility given to individual air carriers.

Practical air traffic flow management has been noted to require “a careful balance between equity and efficiency” (Fearing et al. 2009). Equity in this case refers to ensuring that the costs incurred as a result of ATFM activities do not disproportionately fall on certain airlines or flights. It is worth noting that the twin goals of ATFM are not typically complimentary; there is a “fundamental conflict that may arise between the objectives of efficiency and equity” (Lulli and Odoni 2007). There is a significant gap in the research literature, particularly in the area of network-level air traffic flow management, regarding the lack of consideration of equity as a goal of ATFM activities. It has been suggested that this gap is a key reason past research has not been fully adopted in practice (Gupta and Bertsimas 2010).

There have been recent efforts to bridge the gap between research and practice

identified above. One group of researchers has developed a ‘fairness metric’ to estimate the difference between a given schedule and the first-scheduled, first-served alternative schedule (Fearing et al. 2009). The authors have gone on to incorporate their fairness metric in a mathematical programming formulation of the air traffic flow management problem. Terms penalizing unfair outcomes are weighted and added to the traditional delay cost objective function of air traffic flow management problems. A related research effort proposes an alternate fairness metric, and extends the analysis by allowing airlines to swap landing slots if desired (Gupta and Bertsimas 2010). Again, the work focuses on bringing consideration of equity into a classical air traffic flow management problem formulation. Again a term is added to the delay cost minimizing objective function, weighted to reflect the importance of equity in relation to delay costs.

In this work we reformulate the air traffic flow management problem as a formal biobjective optimization problem. There has not been a formal multiobjective formulation of an air traffic management problem before, to the best of our knowledge, although previous authors have used the language of multiobjective optimization. One paper notes “since there will typically be a trade-off between aggregate system delay and any flight-based fairness criterion, [a new] formulation should essentially consider a bi-criterion approach, enabling the efficient study of the trade-off curve between the two” (Fearing et al. 2009). Another paper notes that in the United States “a primary objective of the [Federal Aviation Administration’s Air Traffic Management] functions is provide fair and equitable access” (Vossen et al. 2010).

There are several reasons why a formal bicriteria approach that treats equity and efficiency objectives separately may be preferable to recently introduced formulations based on objective functions that minimize a weighted summation of different objective functions.

Inefficiency and inequity metrics are fundamentally incompatible, and it’s not clear what a weighted summation of such terms represents. Decision makers must select and then parameterize a model combining various incompatible terms when using a weighted-summation approach. The selection of the ‘optimal’ solution will be very sensitive to the weights used when combining the different objectives, yet decision makers will typically have little confidence in a given set of weights (Ehrgott 2005). Arguably the biggest problem associated with weighted-summation approaches is that such approaches are only able to generate a certain class of ‘optimal’ solutions: those that are found on the boundary of the convex hull of the feasible region of solutions in the multi-dimensional space of the various objective functions (Ehrgott 2005). There exists no intuitive reason for decision makers to restrict themselves to consideration of such solutions. Researchers investigating decision making in complex situations often look for Pareto-optimal policies (Ehrgott 2005). In the context of the air traffic flow management problem, a policy is Pareto optimal if no distinct policy exists that performs better with regards to either equity or efficiency and at least as well with regards to the other objective. The approach introduced in this paper, unlike prior work, is able to identify all Pareto-optimal solutions to the air traffic flow management problem.

## 2 Mathematical formulations

### 2.1 Delay cost minimization

The approach introduced in this paper builds off prior work, particularly the formulation of “the air traffic flow management problem” of (Bertsimas and Patterson 1998). That formulation is introduced here, modified somewhat where helpful. In this formulation, a set of flights  $F$  is to be scheduled so as to avoid local system capacity deficits. Each flight  $f$  in  $F$  has a flight plan consisting of  $N_f$  ordered elements including an origin airport, sections of airspace sectors, and a destination airport. The flight plans are referenced two separate ways. The unordered set  $\rho_f$  includes the  $N_f$  elements in flight  $f$ 's flight plan, while the function  $P$  is used to keep track of the trajectory of the aircraft. For any flight  $f$ ,  $P(f, 1)$  evaluates to the flight's origin airport,  $P(f, N_f)$  the destination airport, and  $P(f, n)$  terms (for values of  $n$  which are integers between 1 and  $N_f$ ) the airspace sectors the flight will travel through arranged in the order with which the sections will be flown through.

Capacities are described here by first discretizing time and then noting the numbers of aircraft that can land at and take off from each airport, as well as fly through each airspace sector, in discrete time slices. Note that the formulation is ideal for considering problems like fog reducing airport throughput at San Francisco International Airport during certain (somewhat predictable) hours of the morning. Let  $\mathcal{T}$  be the set of all time slices considered,  $\mathcal{A}$  the set of all airport considered, and  $\mathcal{S}$  the set of all sectors considered. For any airport  $k$  in  $\mathcal{A}$  and time slice  $t$  in  $\mathcal{T}$ ,  $D_{k,t}$  and  $A_{k,t}$  are defined as the airport departure and arrival capacities. Similarly  $S_{k,t}$  is the capacity of sector  $k$  ( $k \in \mathcal{S}$ ) during time  $t$  ( $t \in \mathcal{T}$ ). Let  $T_{f,k}$  be the set of times when flight  $f$  may be scheduled to depart from, fly through, or land at  $k$  when  $k$  is flight  $f$ 's origin airport, a sector within  $f$ 's flight plan, or  $f$ 's destination airport, respectively.

The formulation of (Bertsimas and Patterson 1998) is innovative in its definition of decision variables.  $x_{f,k,t}$  terms are binary decision variables that are to take on a value of 1 if and only if flight  $f$  in  $F$  has departed from/flown through/arrived at origin airport/airspace sector/destination airport  $k$  before the end of time slice  $t$ . Certain dummy variables are helpful when setting up the problem. For all flights  $f$  in  $F$  and for all  $k$  in  $\rho_f$ ,  $x_{f,k,t}$  terms are set to 0 for all  $t \leq \min T_{f,k} - 1$  and set to 1 for all  $t \geq \max T_{f,k}$ . Given these definitions, the expression  $x_{f,k,t} - x_{f,k,t-1}$  is 1 if and only if flight  $f$  has departed from / flown through / arrived at  $k$  during time slice  $t$ . Similarly, the expression  $x_{f,P(f,n),t} - x_{f,P(f,n+1),t}$  is 1 if and only if flight  $f$  is in/at  $P(f, n)$  during time slice  $t$ . Furthermore, the expression  $\sum_{t \in T_{f,k}} t(x_{f,k,t} - x_{f,k,t-1})$  will yield the time slice when flight  $f$  has departed from / flown through / arrived at  $k$ .

Let  $c_f^g$  be the cost of delaying flight  $f$  on the ground (before the flight takes off) per discrete unit of time. Let  $c_f^a$  be the unit cost of delaying flight  $f$  once it is in the air. Assume the scheduled (and earliest possible) departure and arrival times of flight  $f$  are given as  $d_f$  and  $a_f$ . Then the total cost incurred holding aircraft at origin airports is  $\sum_{f \in F} c_f^g \left[ \sum_{t \in T_{f,P(f,1)}} t(x_{f,P(f,1),t} - x_{f,P(f,1),t-1}) - d_f \right]$ . It is a bit trickier to determine the airborne delay cost since it is essential not to (re)count ground de-

lays. The total airborne delay cost is  $\sum_{f \in F} c_f^a \left[ \sum_{t \in T_{f,P(f,N_f)}} t(x_{f,P(f,N_f),t} - x_{f,P(f,N_f),t-1}) - \sum_{t \in T_{f,P(f,1)}} t(x_{f,P(f,1),t} - x_{f,P(f,1),t-1}) - (a_f - d_f) \right]$ . The objective function of the air traffic flow management problem, as defined in (Bertsimas and Patterson 1998), minimizes the sum of delay costs, as in expression (1) below.

$$\min \sum_{f \in F} \left[ c_f^a \left( \sum_{t \in T_{f,P(f,N_f)}} t[x_{f,P(f,N_f),t} - x_{f,P(f,N_f),t-1}] - a_f \right) + (c_f^g - c_f^a) \left( \sum_{t \in T_{f,P(f,1)}} t[x_{f,P(f,1),t} - x_{f,P(f,1),t-1}] - d_f \right) \right] \quad (1)$$

It is worth noting that the above expression references a number of model parameters and dummy variables. Taking out such references actually yields a simpler, and in some ways more intuitive, objective function. The refined objective function, which to this author's knowledge has not appeared in the research literature to date, is shown as expression (2) below.

$$\min \sum_{f \in F} \left[ (-c_f^a) \sum_{t \in T_{f,P(f,N_f)}} x_{f,P(f,N_f),t} + (c_f^a - c_f^g) \sum_{t \in T_{f,P(f,1)}} x_{f,P(f,1),t} \right] \quad (2)$$

Given an initial solution, setting one additional  $x_{f,P(f,N_f),t}$  decision variable to 1 implies reducing  $f$ 's flight time one time unit and thus reduces delay costs by  $c_f^a$ . Setting one additional  $x_{f,P(f,1),t}$  term to 1 implies scheduling flight  $f$  to take off from its origin airport one time unit earlier. If the arrival time remains unchanged, the flight incurs one less unit of time ground delay but one more unit of time airborne delay and total costs go up by  $(c_f^a - c_f^g)$ .

Airport and sector capacity constraints, modified to match this paper's terminology, are presented in expressions (3), (4), and (5) below.

$$\sum_{f:P(f,1)=k} (x_{f,k,t} - x_{f,k,t-1}) \leq D_{k,t} \quad \forall k \in \mathcal{A}, t \in \mathcal{T} \quad (3)$$

$$\sum_{f:P(f,N_f)=k} (x_{f,k,t} - x_{f,k,t-1}) \leq A_{k,t} \quad \forall k \in \mathcal{A}, t \in \mathcal{T} \quad (4)$$

$$\sum_{f:P(f,i)=k, i < N_f} (x_{f,k,t} - x_{f,P(f,i+1),t}) \leq S_{k,t} \quad \forall k \in \mathcal{S}, t \in \mathcal{T} \quad (5)$$

In order for the decision variables to be consistent, connectivity constraints are required. For example, if an  $x_{f,k,t-1}$  term is set to 1, then  $x_{f,k,t}$  must also be set to 1. This is known as connectivity in time as is captured by expression (6) below.

$$x_{f,k,t} - x_{f,k,t-1} \geq 0 \quad \forall f \in F, k \in \rho_f, t \in T_{f,k} \quad (6)$$

Similarly, the variables must be consistent in terms of individual aircraft trajectories. Let  $\beta_{f,k}$  be the minimum number of time units it takes flight  $f$  to pass through  $k$ . Expression (7) below captures connectivity between sectors.

$$x_{f,P(f,i),t} - x_{f,P(f,i-1),t-\beta_{f,P(f,i-1)}} \leq 0 \quad \forall f \in F, 2 \leq i \leq N_f, t \in T_{f,P(f,i)} \quad (7)$$

Individual aircraft will actually fly multiple flights over the course of a day. Delays propagate as the day goes on. It is important to capture this effect to describe a realistic instance of an air traffic flow management problem. Let  $\mathcal{C}$  be the set of all pairs of flights  $(f_1, f_2)$  where an individual aircraft flies flight  $f_2$  immediately following flight  $f_1$ .  $\chi_{f_2}$  is the (given) minimum time it takes to turnaround the aircraft prior to flight  $f_2$ . Then the so-called airport connectivity constraints can be represented as in expression (8).

$$x_{f_2,P(f_2,1),t} - x_{f_1,P(f_1,N_{f_1}),t-\chi_{f_2}} \leq 0 \quad \forall (f_1, f_2) \in \mathcal{C}, t \in T_{f_2,P(f_2,1)} \quad (8)$$

The final constraint that our decision variables be binary is represented by expression (9).

$$x_{f,k,t} \in \{0, 1\} \quad \forall f \in F, k \in \rho_f, t \in T_{f,k} \quad (9)$$

Objective function (2) together with constraint sets (3) through (9) defines the base air traffic flow management problem, as proposed and studied previously (Bertsimas and Patterson 1998).

## 2.2 Inequity minimization

The formulation of the air traffic flow management problem proposed above is here modified to consider a second objective of minimizing inequity. *Ration by Schedule* is, at least in the United States, “the industry accepted notion of fairness, endorsed by the primary stakeholders, i.e., the [Federal Aviation Administration] and the airlines” (Fearing et al. 2009). Thus, here different schedules are evaluated in terms of how much they deviate from an RBS ideal.

Let’s begin by focusing on a situation where arrival throughput at airports is the major concern, as is common in the United States. Let  $\mathcal{R}$  be the set of all ordered pairs of flights  $(f_1, f_2)$  where  $f_1$  and  $f_2$  are destined for the same airport with  $f_1$  initially scheduled to arrive before  $f_2$ .  $r_{f_1,f_2}$  terms are binary decision variables which are to be set to 1 if and only if  $f_2$  arrives before  $f_1$  in the schedule obtained when solving the air traffic flow management problem (the schedule implied by  $x_{f,k,t}$  terms). In other words,  $r_{f_1,f_2}$  capture *reversals* in the schedule. Such variables were previously proposed (Gupta and Bertsimas 2010), but here such variables are incorporated into separate objective functions for the first time. One example objective function minimizing inequity is shown in expression (10).

$$\min \sum_{(f_1,f_2) \in \mathcal{R}} r_{f_1,f_2} \quad (10)$$

Note that we are counting the number of reversals in order to measure the deviance from an ideal RBS option.

In order to ensure the new decision variables take on values consistent with their desired interpretation, it is necessary to add a constraint set of the formulation. For any pair of flights  $(f_1, f_2)$  in  $\mathcal{R}$ , if  $f_2$  lands before  $f_1$  then  $r_{f_1,f_2}$  must be 1. This

yields expression (11), which was previously proposed (Gupta and Bertsimas 2010).

$$x_{f_2, P(f_2, N_{f_2}), t} - x_{f_1, P(f_1, N_{f_1}), t} - r_{f_1, f_2} \leq 0 \quad \forall (f_1, f_2) \in \mathcal{R}, t \in T_{f_2, P(f_2, N_{f_2})} \quad (11)$$

If there are important constraints on capacity within airspace sectors, it makes some sense to generalize the definition of a reversal. Such a generalization is easily accomplished. Let  $\mathcal{R}'$  be the set of triples  $(f_1, f_2, k)$  where flights  $f_1$  and  $f_2$  are to fly through  $k$  (an airport or airspace sector) with  $f_1$  initially scheduled to arrive before  $f_2$ .  $r'_{f_1, f_2, k}$  terms generalize the previously introduced  $r_{f_1, f_2}$  terms. The objective function minimizing inequity and the constraint keeping decision variables consistent are formulated as in expressions (12) and (13).

$$\min \sum_{(f_1, f_2, k) \in \mathcal{R}'} r'_{f_1, f_2, k} \quad (12)$$

$$x_{f_2, k, t} - x_{f_1, k, t} - r'_{f_1, f_2, k} \leq 0 \quad \forall (f_1, f_2, k) \in \mathcal{R}', t \in T_{f_2, k} \quad (13)$$

Previous authors have noted that applying RBS concurrently for multiple air transportation system resources can yield inefficient results (Fearing et al. 2009). For instance, consider an example where a sizable portion of the flights passing through one airspace sector are destined for a severely capacity constrained destination airport. Forcing all aircraft to go through the sector in the originally scheduled order ensures the sector throughput is reduced to reflect constraints at the troublesome airport. Using a biobjective approach to the air traffic flow management problem allows decision makers to gain greater insight into the trade-offs between efficiency and fairness (in the RBS sense) for particular problem instances.

The metrics described here (from Gupta and Bertsimas, 2010) are not perfect. They count the number of reversals in a schedule but do not take into account the magnitudes of the delays being assigned. Note that a reversal at a busy airport may result in an aircraft being delayed as little as two or three minutes, while another reversal in an infrequently used section of airspace may result in hours of delay. In addition, no accounting is made of the distribution of reversals / delay across the sets of flights and air carriers being managed. Our motivation for considering equity noted the importance of ensuing costs are spread relatively evenly amongst flights and air carriers. An entropy-based objective function could make sense here.

Further research to develop alternate equity maximizing objective functions for the air traffic flow management problem is warranted. More generally, multi-objective optimization could prove quite useful for considering factors rarely mentioned in the current air traffic flow management literature. In particular, it would be possible to develop metrics specifically focused on noise or pollutant emissions and minimize these without having to reduce everything to monetary costs using questionable models or conversion factors.

### 3 Solving the biobjective problem

The  $\epsilon$ -constraint method is arguably the best-known approach for solving multi-objective problems (Ehrgott 2005) and is used here. The approach was originally

introduced by (Haimes, Ladson, and Wismer 1971). Here an objective functions minimizing inequity, expression (12) above, is converted to yield a constraint, expression (14) below.

$$\sum_{(f_1, f_2, k) \in \mathcal{R}'} r'_{f_1, f_2, k} \leq \epsilon \tag{14}$$

Objective function (2) with constraints identified by expressions (3) through (9), (13), and (14) constitutes a typical (single-objective) air traffic flow management problem which can be solved in a reasonable amount of time for realistic problem instances. Varying  $\epsilon$  in expression (14) allows us to find all Pareto-optimal solutions for the biobjective air traffic flow management problem (Ehrgott 2005).

For this particular problem, the inequity measure is a count of the number of reversals. For Pareto-optimal solutions, this metric must take on integer values between 0 and the number of reversals generated by maximizing efficiency and ignoring inequity. Note that each reversal necessarily involves delaying the aircraft originally scheduled to access the shared resource first. Thus, there is some reason to be optimistic that there will not be an unreasonably large number of reversals in an efficiency-maximizing schedule. We start by finding the efficiency-maximizing schedule. We then take the number of reversals in this schedule, subtract one, and set  $\epsilon$  equal to this value in expression (14). We maximize efficiency alone, with the addition of constraint (14). The result is another Pareto-optimal schedule. We then repeat the process, counting the number of reversals in the latest schedule, subtracting one, setting  $\epsilon$  equal to this value, and resolving. We stop when the problem becomes infeasible or we reach a situation where a schedule involving no reversals is generated. Along the way, we have found all the Pareto-optimal solutions to the biobjective air traffic flow management problem.

In order to test the defined algorithm, we have run computational studies with randomly generated biobjective air traffic flow management problems. In the generated problems, there were 20 airports, 200 airspace sectors, 168 discrete periods of time, and each flight was assumed to fly through 5 airspace sectors between its origin and destination airport with flight delays of between 0 and 6 time periods considered feasible. The chosen parameter values were taken from (Bertsimas and Patterson 1998), which describes the values as typical for realistic-sized problem instances. 2,000 flight paths were randomly generated, along with changing airport and airspace sector capacity constraints.

Figure 1 gives an example of obtained results, focusing on the trade-off between the efficiency and equity objectives. The x-axis shows the number of reversals in obtained solutions, while the y-axis shows the value of the delay minimization objective function. It is worth noting that the reformulation of the objective function used here, expression (2) introduced above, yields negative delay costs. Obtained values can be interpreted as the difference in delay costs between a given solution and a worst-case / maximum delay solution.

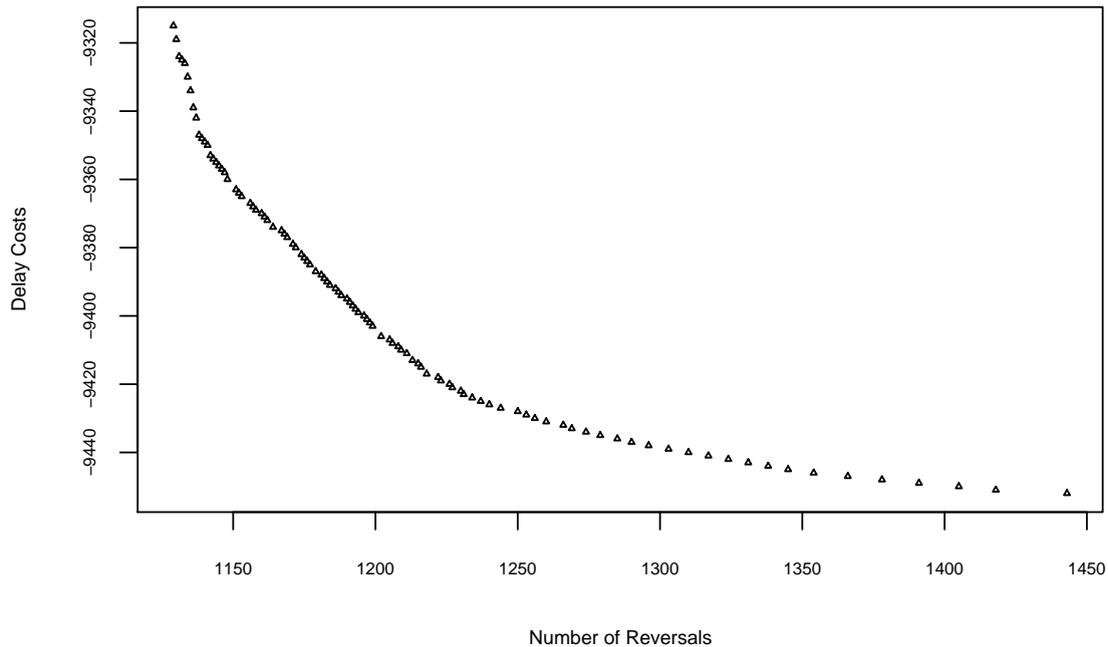


Figure 1: The trade-off between efficiency and equity.

Figure 1 makes clear that there is a choice to be made between efficiency and equity when scheduling flights. The general shape of the relationship shown is convex, indicating that focusing on only one of the objectives may yield very poor performance with regards to the other objective. Although the general shape of the relationship is convex, there are some points that would not be on the boundary of the convex hull of feasible points. In other words, some Pareto-optimal schedules were found that would not have been found using an approach minimizing a weighted summation of delay and inequity costs.

## 4 Conclusion

The air traffic flow management problem was here extended to a multi-objective optimization problem minimizing inefficiency and inequity. This is an important contribution given that past researchers have identified the failure of past formulations to consider equity concerns as the primary reason prior research results have not been adopted in practice in air traffic control. Computational studies show realistic sized biobjective air traffic flow management problems can be solved in reasonable amounts of time, and yield Pareto-optimal solutions not found using distinct approaches based on minimizing a weighted summation of delay and inequity costs. Further work is warranted to define additional objective functions for air traffic flow management problems, and to devise strategies for efficiently solving such problems. In particular, environmental concerns would be worth investigating. It would also be interesting to consider stochastic formulations to address the issue of uncertainty in airspace and airport capacity estimates, or to consider formulations that allow for dynamic rerouting of aircraft.

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