Cost-Recovering, Revenue-Adequate Single Settlement Schemes for Electricity Markets

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Abstract

The penetration of intermittent renewable sources in electricity markets is increasing. This intermittency increases the uncertainty in electricity markets, and the amount of expensive re-adjustment required from other generators when renewables deviate significantly from their forecast capacity. Dispatching generators according to the optimal solution of a stochastic program minimizes the generation cost plus the expected cost of deviating. The first and second stages of this stochastic program correspond to policies for day-ahead and real-time dispatches under different realisations of a random variable. We derive a single-settlement, discriminatory payment scheme which is revenue adequate in expectation and provides generator cost recovery in every scenario. We discuss the attributes of this settlement scheme, and compare it with other schemes in the literature, in both theoretical terms and in the context of the New Zealand Electricity Market under April 2015 conditions.

Keywords: Electricity Markets, Stochastic Programming, Wind Power

1. Introduction

In response to increasing pressure to reduce CO2 emissions, electricity markets are seeing increasing levels of generation from intermittent renewable sources such as wind and solar. As can be seen in the South Australian and German electricity markets, high levels of penetration of renewables can create difficulties for the system operator when dispatching generators. These difficulties arise because thermal generators need to be informed how much they are required to generate, before the generation output of renewables is known.

Traditionally, this problem is solved by dispatching generators according to the forecast generation output of renewables. A frequency-keeping station and reserves are then used to take corrective actions. However, this is suboptimal, as it does not consider the cost of the corrective action when dispatching generators.

Instead, we follow the methodology in [2][3][4][6][7], by explicitly modelling the uncertainty in the generation capacity of renewables as a random variable, and supplementing reserves with a real-time electricity market, which is cleared after the generation capacity of all renewables is realised. This is achieved by converting the market clearing problem to a two-stage stochastic program.

The first-stage or day-ahead dispatch, x, is the dispatch that generators are advised to prepare for, based on the ISO's probabilistic forecast of the generation output of all renewables at gate closure. This dispatch is necessary, because inflexible thermal generators require advanced notice for the amount of electricity they are required to generate, and because other market participants may prefer to have certainty in their dispatch.

The random outcome, ω, represents a scenario which corresponds to a particular realisation of a random variable, which prescribes the generation output of all intermittent generators[1] and occurs with probability $P(\omega)$.

The second-stage or real-time dispatch, $X(\omega)$, is then the dispatch that other generators produce. It is announced in real-time, after the generation output of all renewable generators is realised.

When non-renewable generators deviate from x to $X(\omega)$, they incur a cost. For thermal generators, this cost is incurred in the form of thermal stress and wear-and-tear on generation equipment. For hydro generators, this cost is incurred in the form of an opportunity cost, which occurs when hydro generators are dispatched in a trade period where they are paid a lower marginal price than if they were to be dispatched in a subsequent trade period[2]. In either case, we assume that the cost is a constant for every MW. This is without loss of generality, as generators can offer in multiple tranches.

We proceed to formally define the stochastic program considered here, before deriving a new payment scheme for paying generators and charging consumers.

2. The Stochastic Dispatch Mechanism

2.1. Notation

Let the sets which the decision variables are defined over be as follows:

- i is the index of a generator.
- N is the set of all nodes.
- $j(i)$ is the node j where generator i is located.
- $T(n)$ is the set of all offer tranches from all generators at node n.

Let the decision variables and problem data be defined as follows:

[1] As opposed to a single generator, to avoid modelling correlations in the system.

[2] In this document, we assume that hydro generators have access to their expected opportunity cost, so that the resulting Stochastic Dispatch Mechanism can be solved as a linear program.

- c is the offer price for generating one MW of electricity for each generator.
- x is the day-ahead dispatch, which is announced at gate closure.
- $X(\omega)$ is the real-time dispatch in scenario ω.
- $G(\omega)$ is the maximum output of all generators in scenario ω.
- $u(\omega)$ and $v(\omega)$ are the amounts which each generator deviates up/down by in scenario ω. That is, $u(\omega) =\max(X(\omega) - x, 0)$, and $v(\omega) =\max(x - X(\omega), 0)$.
- r_u and r_v are the costs of deviating up or down by one MW.
- U is a set containing all the feasible flows in the network. We assume that $0 \in U$, because demand is allowed to be shed at a price of VOLL.
- $F(\omega) \in U$ is the flow through the network in scenario ω.
- $\tau_n(F(\omega))$ is the net injection from the grid into node n in scenario ω.
- $d_n(\omega)$ is the consumer demand when the intermittent generation output is ω.

2.2. The Stochastic Dispatch Mechanism (SDM)

The day-ahead dispatch is found by solving the following stochastic linear program for x, as defined in [6]:

$$\begin{aligned}
\text{SLP: min} \quad & E_\omega[c^\top X(\omega) + r_u^\top u(\omega) + r_v^\top v(\omega)] & & \\
\text{s.t.} \quad & \sum_{i \in T(n)} X_i(\omega) + \tau_n(F(\omega)) = d_n(\omega), & & \forall \omega \in \Omega,\ [P(\omega)\lambda_n(\omega)], \\
& x + u(\omega) - v(\omega) = X(\omega), & & \forall \omega \in \Omega,\ [P(\omega)\rho(\omega)], \\
& F(\omega) \in U, & & \forall \omega \in \Omega, \\
& 0 \leq X(\omega) \leq G(\omega),\ u(\omega), v(\omega) \geq 0, & & \forall \omega \in \Omega.
\end{aligned}$$

After the optimal day-ahead dispatch x^* is found, the intermittent generation output $\omega = \hat{\omega}$ is realised, and the ISO solves the following recourse LP for $X(\hat{\omega})$, to determine how generators should be re-dispatched:

$$\begin{aligned}
\text{min} \quad & c^\top X(\hat{\omega}) + r_u^\top u(\hat{\omega}) + r_v^\top v(\hat{\omega}) & & \\
\text{s.t.} \quad & \sum_{i \in T(n)} X_i(\hat{\omega}) + \tau_n(F(\hat{\omega})) = d_n(\hat{\omega}), & & [\lambda_n(\hat{\omega})], \\
& x^* + u(\hat{\omega}) - v(\hat{\omega}) = X(\hat{\omega}), & & [\rho(\hat{\omega})], \\
& F(\hat{\omega}) \in U, & & \\
& 0 \leq X(\hat{\omega}) \leq G(\hat{\omega}),\ u(\hat{\omega}), v(\hat{\omega}) \geq 0. & &
\end{aligned}$$

2.3. Revenue Adequacy and Cost Recovery

Definition 2.1. A market clearing mechanism is *revenue adequate* if and only if in every scenario $\omega \in \Omega$, clearing the market does not leave the system operator in a financial deficit. That is,

$$\sum_n \lambda_n(\omega)\tau_n(F(\omega)) \geq 0,\ \forall \omega \in \Omega.$$

Revenue adequacy is desirable, as it ensures that the ISO is never out of pocket.

Definition 2.2. A market clearing mechanism exhibits *short-run cost recovery* if and only if in every scenario $\omega \in \Omega$, all bidders recover their short-run (fuel and deviation) costs. That is,

$$P_i(\omega) - c_i X_i(\omega) - r_{u,i} u_i(\omega) - r_{v,i} v_i(\omega) \geq 0, \quad \forall i, \quad \forall \omega \in \Omega,$$

where $P_i(\omega)$ is the amount paid to generator i by the ISO in scenario ω.

If a market clearing mechanism exhibits *revenue adequacy in expectation* then the ISO will not be out of pocket in the long-run. Similarly, if a market clearing mechanism exhibits *expected short-run cost recovery* then all generators will recover their generation and ramping costs in the long-run[3].

3. A New Pricing Mechanism

We introduce the following terms:

- Let $\delta_{in} = 1$ when agent i is located at node n and 0 otherwise.
- Let V be the value of lost load (VOLL).
- Let $\bar{\rho} = E_\omega[\rho(\omega)]$ be the expected value of the dual variable $\rho(\omega)$ in the SLP.
- Let $\bar{\lambda} = E_\omega[\lambda(\omega)]$ be the expected value of the dual variable $\lambda(\omega)$ in the SLP.

Two existing single settlement payment mechanisms in the literature are:

- Paying $(\bar{\lambda}_{j(i)} - \lambda_{j(i)}(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega)$ to generator i, as proposed in [4]. This results in revenue adequacy in expectation and short-run cost recovery in expectation. We refer to this as the PZP mechanism.
- Paying $\lambda_{j(i)}(\omega)X_i^*(\omega)$ to generator i, as proposed in [6]. This results in revenue adequacy in each scenario and cost recovery in expectation. We refer to this as the ZPBB mechanism.

Provided that both mechanisms implement the optimal solution to SLP, the ZPBB and PZP payment mechanisms are equal in expectation (as $E[\bar{\lambda}] = \bar{\lambda} = E[\lambda]$). Therefore, since the ZPBB mechanism provides cost recovery in expectation, it follows that the PZP mechanism also provides cost recovery in expectation.

A different payment mechanism can be derived by taking the Lagrangian of SLP. We are examining the Lagrangian, because a convex program solved to optimality by a central planner yields the same dispatch as a competitive market equilibrium in which agents[4] maximise profits at prices defined by dual variables. The

[3] As opposed to *long-run cost recovery*, which also includes fixed plant costs.

[4] Here agents aim to maximise their expected profit and act as price takers.

Lagrangian therefore allows us to consider the profit of an arbitrary agent under various payment mechanisms.

The Lagrangian $\mathscr{L}$ for SLP is:

$$\mathscr{L} = \sum_i c_i x_i + \sum_\omega P(\omega) \sum_i \big(r_{u,i} u_i(\omega) + r_{v,i} v_i(\omega)\big) - \sum_n V \sum_\omega P(\omega) d_n(\omega)$$
$$+ \sum_n \sum_\omega P(\omega)\big(d_n(\omega) - \sum_i \delta_{in} X_i(\omega) - \tau_n(F(\omega))\big)\lambda_n(\omega)$$
$$+ \sum_\omega P(\omega) \sum_i \rho_i(\omega)\big(v_i(\omega) - u_i(\omega) + X_i(\omega) - x_i\big),$$

which is to be minimised, subject to the following constraints:

$$0 \leq X_i(\omega) \leq G_i(\omega),\ u_i(\omega), v_i(\omega), x_i \geq 0,$$
$$F(\omega) \in U.$$

All Lagrange multipliers should be nonnegative. Observe that $\sum_n \delta_{in}\lambda_n(\omega) = \lambda_{j(i)}(\omega)$. Now, rearranging to *maximise* by supply agent i and demand agent n, yields:

$$\max \sum_i (-c_i + \bar{\rho}_i) x_i + \sum_n \sum_\omega P(\omega)\left(V - \lambda_n(\omega)\right) d_n(\omega)$$
$$+ \sum_i \sum_\omega P(\omega)\left(\lambda_{j(i)}(\omega) - \rho_i(\omega)\right) X_i(\omega) + \sum_n \sum_\omega P(\omega)\lambda_n(\omega)\tau_n(F(\omega))$$
$$+ \sum_i \sum_\omega P(\omega)\big(\rho_i(\omega) - r_{u,i}\big)u_i(\omega) + \sum_i \sum_\omega P(\omega)\big(-\rho_i(\omega) + r_{v,i}\big)v_i(\omega)$$
$$\text{s.t. } 0 \leq X_i(\omega) \leq G_i(\omega),\ u_i(\omega), v_i(\omega), x_i \geq 0,$$
$$F(\omega) \in U.$$

This can be decoupled by supply agent. Agent i solves the following stochastic program for x_i in the first stage:

$$\text{P}(i):\ \max \sum_\omega P(\omega)\Big((\bar{\rho}_i - c_i)x_i + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i(\omega)$$
$$+(\rho_i(\omega) - r_{u,i})u_i(\omega) + (-\rho_i(\omega) - r_{v,i})v_i(\omega)\Big)$$
$$\text{s.t. } 0 \leq X_i(\omega) \leq G_i(\omega),\ u_i(\omega), v_i(\omega), x_i \geq 0.$$

Note that x_i is a here-and-now decision, $X_i(\omega)$ is a wait-and-see decision and we have the following result:

Lemma 1. *For every generation agent i, the optimal dispatch x_i^* satisfies $(\bar{\rho}_i - c_i)x_i^* = 0$ (i.e. $c_i = \bar{\rho}_i$ for all agents i which have a non-zero first stage dispatch).*

Proof. If $\bar{\rho}_i > c_i$ then x_i^* being unbounded from above could be chosen to make the Lagrangian $\mathscr{L} = -\infty$. It follows that $\bar{\rho}_i \leq c_i$ and x_i^* is such that $(\rho_i - c_i)x_i^* = 0$. □

We now consider the real-time dispatch, $X_i(\omega)$, to yield the following result:

Lemma 2. *For every generation agent i with $X_i^*(\omega) > 0$, it follows that $\rho_i(\omega) \leq \lambda_{j(i)}(\omega)$.*

Proof. We take the contrapositive. Suppose $\rho_i(\omega) > \lambda_{j(i)}(\omega)$. Then since $X_i(\omega) \geq 0$ it follows that the optimal choice of $X_i(\omega)$ is $X_i^*(\omega) = 0$. □

We now consider the optimal real-time deviations $u_i^*(\omega)$ and $v_i^*(\omega)$ to yield:

Lemma 3. *In the Lagrangian $\mathscr{L}$, the optimal choices of u_i^* and v_i^* must satisfy:*

$$(\rho_i(\omega) - r_{u,i})u_i^*(\omega) + (-\rho_i(\omega) - r_{v,i})v_i^*(\omega) = 0.$$

Proof. Suppose not. Then since $\mathscr{L}$ has the constraint $u_i(\omega), v_i(\omega) \geq 0$, it follows that $(\rho_i(\omega) - r_{u,i}) > 0$ or $(-\rho_i(\omega) - r_{v,i}) > 0$. In either case, $u_i(\omega), v_i(\omega)$ can be chosen so as to make the Lagrangian $\mathscr{L} = -\infty$. □

We now use Lemmas 1, 2 and 3 to yield the following result:

Lemma 4. *For every i, if agent i has made the optimal choice of x_i^* then paying:*

$$\bar{\rho}_i x_i^* + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i^*(\omega) + \rho_i(\omega)u_i^*(\omega) - \rho_i(\omega)v_i^*(\omega),$$

results in cost recovery in every scenario.

Proof. Assume that agent i has seen the realisation of ω (i.e. agent i is making its second stage decision). Then agent i's optimization problem is:

$$\begin{aligned}\phi_i(\omega) = \max_{X_i(\omega)} \; & (\bar{\rho}_i - c_i)x_i^* + (\lambda_{j(i)} - \rho_i(\omega))X_i(\omega) \\ & + (\rho_i(\omega) - r_{u,i})u_i(\omega) + (-\rho_i(\omega) - r_{v,i})v_i(\omega) \\ & \text{s.t. } 0 \leq X_i(\omega) \leq G_i(\omega),\; u_i(\omega), v_i(\omega) \geq 0.\end{aligned}$$

Now, by lemma 3, we know that:

$$(\rho_i(\omega) - r_{u,i})u_i^*(\omega) + (-\rho_i(\omega) - r_{v,i})v_i^*(\omega) = 0.$$

Therefore, the optimal profit of agent i in scenario ω satisfies :

$$\phi_i^*(\omega) = (\bar{\rho}_i - c_i)x_i^* + (\lambda_{j(i)} - \rho_i(\omega))X_i^*(\omega).$$

We know from Lemma 1 that $(\bar{\rho}_i - c_i)x_i^* = 0$, and we know from Lemma 2 that $(\lambda_{j(i)} - \rho_i(\omega))X_i^*(\omega) \geq 0$. Therefore, $\phi_i^*(\omega) \geq 0 \;\; \forall \omega \in \Omega$. □

We now wish to simplify the payment mechanism. To do so, we invoke:

Lemma 5. *For every i, if agent i makes the optimal choice of x_i^* and $X_i^*(\omega)$ then paying:*

$$\bar{\rho}_i x_i^* + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i^*(\omega) + \rho_i(\omega)u_i^*(\omega) - \rho_i(\omega)v_i^*(\omega),$$

gives the same profit as paying agent i:

$$(\bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega).$$

Proof. SLP has the constraint:

$$x + u(\omega) - v(\omega) = X(\omega), \ \ \forall \omega \in \Omega, \ [P(\omega)\rho(\omega)].$$

The result follows by complementarity. □

We refer to paying $\bar{\rho}_i - \rho_i(\omega)x_i + \lambda_{j(i)}(\omega)X_i(\omega)$ to generator i and charging $\lambda_n(\omega)d_n(\omega)$ to demand agent n as the CWPZ payment mechanism. By complementarity, the CWPZ mechanism also exhibits short-run cost recovery.

We can write the profit earned by agent i under the CWPZ scheme as:

$$(\bar{\rho}_i - \rho_i(\omega) - c_i)x_i + \lambda_{j(i)}X_i(\omega) - r_{u,i}u_i(\omega) - r_{v,i}v_i(\omega)$$

Taking the expectation of $\phi_i(\omega)$ with respect to ω then yields:

Lemma 6. *If $(x^*, X^*(\omega))$ solves SLP, then paying $(\bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega)$ to generator i results in the same expected profit for generator i as the ZPBB payment mechanism.*

Proof. Under the CWPZ mechanism, the expected profit for generator i is:

$$E_\omega\Big[(-c_i + \bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega) - r_{u,i}u_i^*(\omega) - r_{v,i}v_i^*(\omega)\Big]$$
$$= E_\omega\Big[- c_i x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega) - r_{u,i}u_i^*(\omega) - r_{v,i}v_i^*(\omega)\Big],$$

which is the expected profit obtained from being paid $\lambda_{j(i)}(\omega)X_i(\omega)$. Therefore, even though the CWPZ mechanism is discriminatory, it results in the same expected profit in the first stage as the non-discriminatory ZPBB mechanism. □

We now wish to show that the CWPZ mechanism gives revenue adequacy in expectation. To do so, we invoke:

Lemma 7. *If $(x^*, X^*(\omega))$ solves SLP, then paying each agent $(\bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega)$ results in revenue adequacy in expectation.*

Proof. Under the CWPZ payment scheme, the ISO's expected profit is:

$$E_\omega\Big[\sum_n \lambda_n(\omega)(d_n(\omega) - \sum_{i \in T(n)} X_i^*(\omega)) - (\bar{\rho} - \rho(\omega))^\top x^*\Big] = E_\omega\Big[\sum_n \lambda_n(\omega)\big(d_n(\omega) - \sum_{i \in T(n)} X_i^*(\omega)\big)\Big].$$

We invoke complementarity to make the substitution:

$$\lambda_n(\omega)\tau_n(F^*(\omega)) = \lambda_n(\omega)\big(d_n(\omega) - \sum_{i \in T(n)} X_i^*(\omega)\big).$$

After substituting, the ISO's expected profit becomes: $E_\omega\Big[\sum_n \lambda_n(\omega)\tau_n(F^*(\omega))\Big]$. Now, since it is possible for the system to shed demand at the price VOLL, a feasible choice for the ISO is $F(\omega) = 0 \ \ \forall \omega \in \Omega$, which gives an expected profit of 0[5]. As any feasible action gives a lower bound on the ISO's expected profit, it follows that the CWPZ scheme gives revenue adequacy in expectation. □

[5]This is true with concave losses, since $\tau_n(0) = 0$ irrespective of the choice of loss function.

We then use Lemma 7 to yield:

Lemma 8. *A sufficient condition for revenue adequacy in scenario* $\hat{\omega}$ *is:*

$$\left(\rho(\hat{\omega}) - \bar{\rho}\right)^{\top} x^{*} \geq 0.$$

Proof. Suppose that $\left(\rho(\hat{\omega}) - \bar{\rho}\right)^{\top} x^{*} \geq 0$. Then the ISO's profit is:

$$\sum_{n} \lambda_n(\omega)\tau_n(F^{*}(\hat{\omega})) + \left(\rho(\hat{\omega}) - \bar{\rho}\right)^{\top} x^{*} \geq \sum_{n} \lambda_n(\hat{\omega})\tau_n(F^{*}(\hat{\omega})) \geq 0.$$

Now, since choosing $F(\hat{\omega}) = 0$ results in the ISO achieving a profit of 0, it follows that the ISO will be revenue adequate. As a result, we can bound the maximum loss seen by the ISO in any scenario by: $\max_{\omega} \left(\bar{\rho} - \rho(\omega)\right)^{\top} x^{*}$. □

Lemma 8 can readily be extended to yield:

Lemma 9. *A sufficient condition for cost recovery for agent* i *in scenario* $\hat{\omega}$ *under the ZPBB payment mechanism is:* $(\rho_i(\hat{\omega}) - \bar{\rho}_i)x_i^{*} \geq 0$.

Proof. Under the CWPZ payment mechanism, agents are paid $\bar{\rho}_i - \rho_i(\hat{\omega})x_i^{*} + \lambda_{j(i)}(\hat{\omega})X_i^{*}(\hat{\omega})$, which provides cost recovery in every scenario. Therefore, if $(\rho_i(\hat{\omega}) - \bar{\rho}_i)x_i^{*} \geq 0$ then paying $\lambda_{j(i)}(\hat{\omega})X_i^{*}(\hat{\omega})$ provides cost recovery in scenario $\hat{\omega}$. □

We have established that the CWPZ mechanism has cost recovery in every scenario, and revenue adequacy in expectation, in a network with non-negative nodal prices and concave losses. In addition, each agent has the same expected profit under the CWPZ mechanism as under the PZP and ZPBB mechanisms, meaning that risk-neutral price-taking agents are indifferent between the three mechanisms.

The above lemmas depend only on the convexity of the dispatch problem. Therefore, cost recovery and expected revenue adequacy can be extended to include generators that have convex cost functions and convex costs of deviation. However, we have not done this here, in order that the resulting market clearing problems can be solved as linear programs.

4. Benchmarking on the NZEM

The ZPBB scheme outperforms the PZP scheme, as can be seen in Table 1.

Table 1: Attributes of the three payment schemes.

Payment scheme	Revenue adequacy	Cost recovery	Attribute
PZP	In expectation	In expectation	Uniform pricing
ZPBB	In every scenario	In expectation	Uniform pricing
CWPZ	In expectation	In every scenario	Discriminatory

However, it is unclear whether the ZPBB or CWPZ mechanism has the best performance. Therefore, we use the methodology in [1] to compare the performance of both mechanisms in the setting of the New Zealand Electricity Market (NZEM).

The software used to clear the NZEM is Scheduling Pricing and Dispatch (SPD), a vectorised replica of which (vSPD) is publicly available. We convert the GAMS implementation of vSPD to be the deterministic equivalent of SLP, by introducing the set Ω. We then modify the demand and generation data to contain forecast wind output levels (rather than realised output levels), and be defined over Ω. We alter the constraints in vSPD to be defined over the set Ω, and modify the objective function to include the expected cost of deviation.[6]. The resulting stochastic LP gives a two-hour-ahead market clearing problem for the NZEM.

To clear the real-time market, we modify vSPD's objective function to include the cost of deviating from the two-hour-ahead market. As we are back-testing, we use existing generation and demand data, which contain historical wind output levels.

4.1. Modelling wind in the NZEM

The NZEM has 19 operational wind farms, which have a cumulative production capacity of 690 MW[7]. Of these, the Tararua, Te Apati and Te Rere Hau (Central North Island) wind farms have a capacity of 300 MW, and the West Wind and Mill Creek (Wellington) wind farms have a capacity of 200 MW.

To reduce the number of dimensions in our problem, we model the *total* power output in the Central North Island and the *total* power output in Wellington as realisations of random variables. Furthermore, we assume that we have perfect foresight for generation at all other wind farms, and treat this output as deterministic negative demand.

To reduce the complexity of our scenarios, we assume that the deviations in the total wind power in the Central North Island and Wellington are *conditionally* independent, meaning that we can take the product of their marginal probability densities to create the joint density of ω. That is, we assume that:

$$f(\theta_{T+1}^{WGTN}, \theta_{T+1}^{CNI} | \theta_T^{WGTN}, \theta_T^{CNI}) = f_W(\theta_{T+1}^{WGTN} | \theta_T^{WGTN}) f_G(\theta_{T+1}^{CNI} | \theta_T^{CNI}),$$

where θ_T^{WGTN} is the total power in Wellington at time T, normalised over $\theta \in [0, 1]$, f is the joint density function and f_W, f_G are the marginal density functions. Conditional independence is a valid assumption, since the correlation between the bi-hourly deviations in the Central North Island and Wellington was 0.0788 over $2011 - 2014$.

[6] A Benders decomposition of stochastic vSPD was also implemented, but this found to be around 120 times slower at clearing the NZEM when using 25 scenarios.

[7] Production capacity refers to the maximum output of a wind farm under ideal conditions.

We use quantile regression to generate scenarios for the distribution of wind power two hours in the future, conditional on the current wind power level[8]. In particular, we approximate the τth quantile of the conditional distribution of future wind power, θ_{T+1}, by the following function:

$$F^{-1}_{\theta_{T+1}|\theta_T}(\tau) \cong \theta_T + \sum_{n=0}^{n=5} \beta_n \theta_T^n,$$

where we use the R package Quantreg to solve for the appropriate β coefficients. As we are back-testing on the NZEM, we use data from $2011 - 2014$ to generate our quantiles, with a view to test them out-of-sample on data from April 2015.

The two-hour-ahead policy generated from using five scenarios per wind farm is very similar (0.034% different) to the policy from using 15 scenarios per wind farm, and takes two minutes per trade period to generate instead of 60. Therefore, we use the 10th, 30th, 50th, 70th and 90th quantiles from the two sets of wind farms for our scenarios, giving 25 scenarios provided to SLP in total. The quantiles for the Wellington wind farms are depicted in Figure 1, where the 1st, 5th, 95th and 99th quantiles are shown in blue for reference.

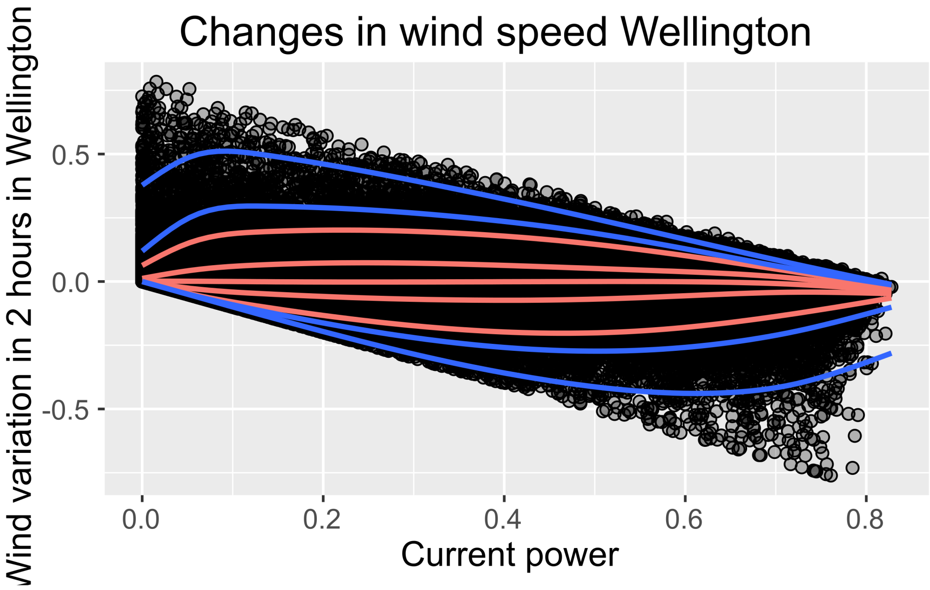

Figure 1: Conditional distribution of wind two hours in the future.

The cost of deviation for each generator is not provided in vSPD. Therefore, we model it as the inverse of the generator ramp rate, by setting $r_{u,i}$ and $r_{v,i}$ to be:

$$r_{u,i} = \frac{1}{\text{Ramp up rate}}, \; r_{v,i} = \frac{1}{\text{Ramp down rate}}.$$

[8]We choose two hour gate closure as this is currently used by the NZEM participants.

This is a reasonable approximation, since generators with a higher ramp rate are re-dispatched first, which is a merit order re-dispatch.

4.2. Revenue Adequacy

As summarised in Table 2, whilst the CWPZ mechanism only provides revenue adequacy in expectation, the loss incurred by the ISO from this defect is insignificant compared to the magnitude of the ISO's expected profit[9].

Table 2: Revenue adequacy properties on the NZEM.

Payment scheme	ZPBB	CWPZ	Units
Expected ISO shortfall	0	45	\$ NZD per year
Expected ISO profit	$66,898,000$	$66,853,000$	\$ NZD per year

Therefore, we do not consider revenue adequacy any further in the context of the NZEM, as it is unimportant for benchmarking. However, revenue adequacy could be an issue for a market with a different topology, such as Pennsylvania-New Jersey-Maryland (PJM). For such a market, the sampling scheme used could impact the expected ISO shortfall significantly. We leave this issue as future work.

4.3. Cost Recovery

For benchmarking purposes, we consider cost recovery by tranche rather than by generator, since we are assuming that the NZEM is a perfectly competitive market and all generators are bidding at marginal cost. By summing the total revenue shortfall experienced by generators, we obtain Table 3. Note that the average shortfall per trade period considers the situation where uplift payments are immediately provided to generators that fail to recover their costs. Aggregating settlement periods would cause the average shortfall under ZPBB to tend towards the average shortfall under CWPZ, by the law of large numbers.

Table 3: Generator Revenue Shortfall on the NZEM.

Payment scheme	ZPBB	CWPZ	Units
Lemma 2 applies	$28,400$	0	\$ NZD per year

Therefore, in a perfectly competitive version of the NZEM, the CWPZ scheme saves the ISO around $\$28,400$ per year compared to the ZPBB mechanism, if the ISO is required to provide uplift payments to all generators that fail to recover their costs. That is, with perfect competition and the sampling scheme used here, the CWPZ scheme outperforms the ZPBB scheme in the NZEM.

[9]Expected shortfall refers to the expected loss the ISO encounters in trade periods where it does not achieve revenue adequacy, multiplied by the probability of failing to achieve revenue adequacy.

5. Conclusions

The CWPZ mechanism outperforms the ZPBB mechanism on a perfectly competitive version of the NZEM with a quantile regression based sampling scheme. However, it is unclear which mechanism is best suited for a market containing a finite number of market participants, some of whom exercise market power.

It is also unclear if the advantages of the CWPZ mechanism are outweighed by the fact that it is discriminatory. As noted in [5], a uniform price is thought to be necessary to encourage investment in an electricity market. Without uniform pricing, generation units may fail to recover their long-run marginal costs, which could cause shortages in times of scarcity. However, discussion of this topic is beyond the scope of this paper.

Market designers therefore need to choose between mechanisms which require more uplift payments and have uniform pricing, and mechanisms which require fewer uplift payments but are discriminatory.

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