Multi-level modelling with missing data for recovering the history of geothermal wells

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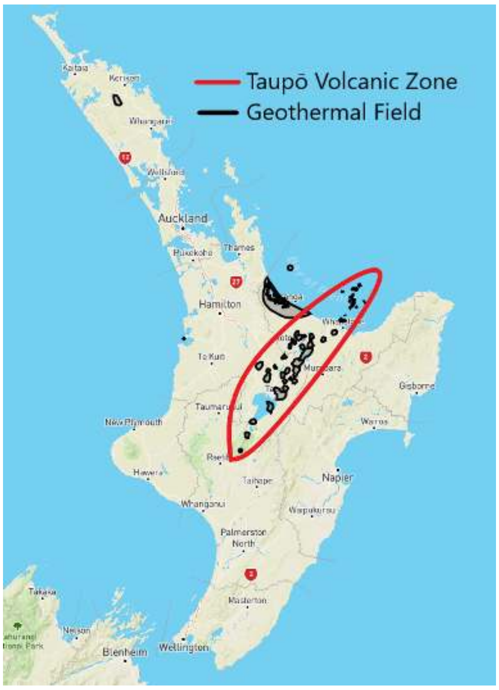
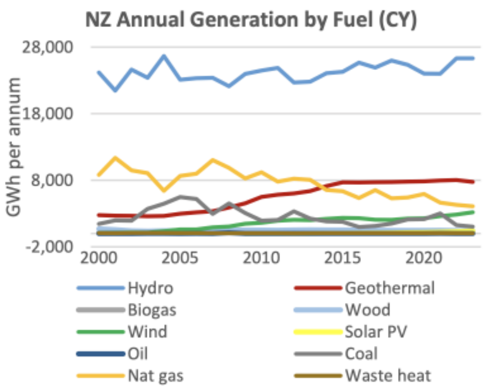
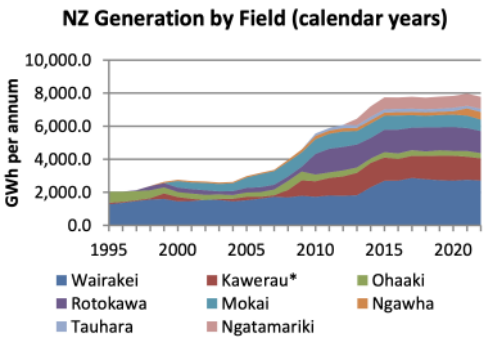
Geothermal Energy in New Zealand

Comprises 17.8% of New Zealand's total power generation in 2023.

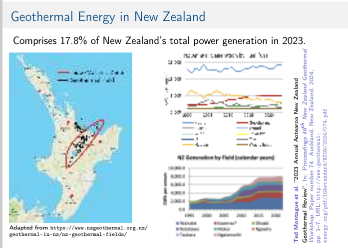


Adapted from `https://www.nzgeothermal.org.nz/geothermal-in-nz/nz-geothermal-fields/`





Ted Montague et al. "2023 Annual Aotearoa New Zealand Geothermal Review". In: *Proceedings 46th New Zealand Geothermal Workshop*. Paper number 74. Auckland, New Zealand, 2024, pp. 1–7. URL: `http://www.geothermal-energy.org/pdf/IGAstandard/NZGW/2024/074.pdf`

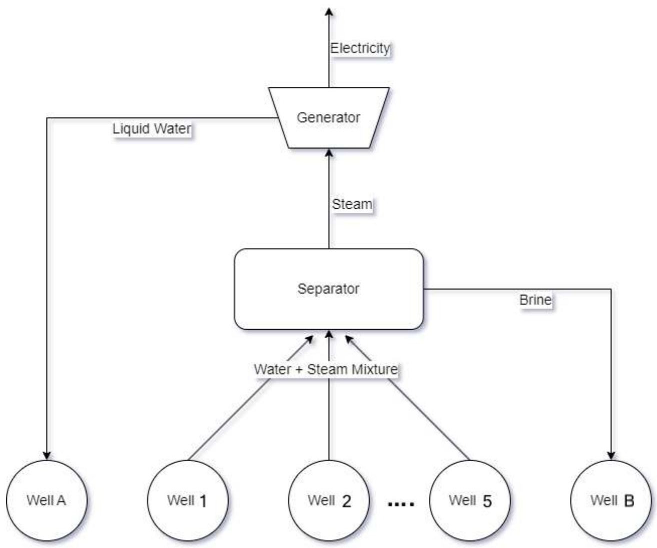


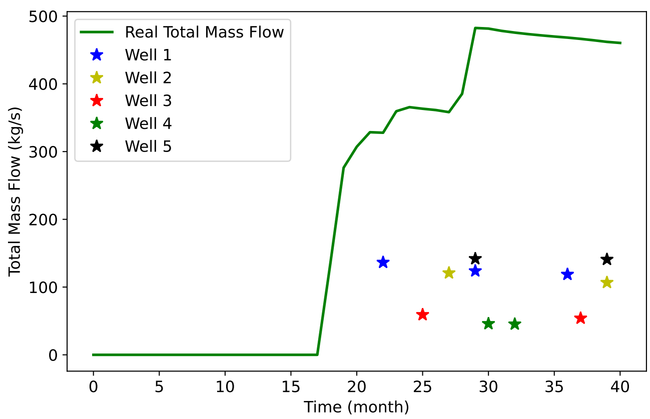
Notes

- Geothermal reservoirs hold water (in liquid and gaseous forms) at high temperatures and pressures in underground volumes of permeable rock.
- This hot steam can be extracted to provide physical heat to a town or broader area, or it can be used to generate electrical energy by driving large steam turbines.
- The majority of NZ's Geothermal reservours lie in the Taupo Volcanic Zone, which runs Southwest from Whakaari (White Island) through Lake Taupo, to Mount Ruapeh.
- It is important not to extract too much mass from a geothermal reservoir because over-extraction may reduce its active lifespan.
- Geothermal reservoir modeling is used to monitor the health of geothermal reservoirs and predict the future state of geothermal fields by being calibrated to historical behaviour.

Sparse measurements

- Total mass flow into separator is measured frequently.
- Mass flow of individual wells is measured sparsely.





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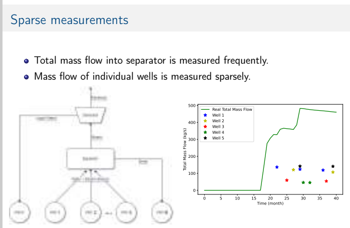
└─Geothermal Energy in NZ
└─Sparse measurements

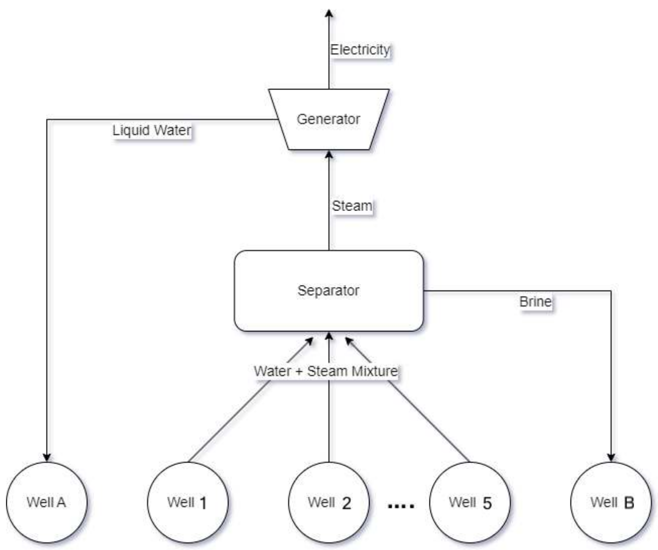


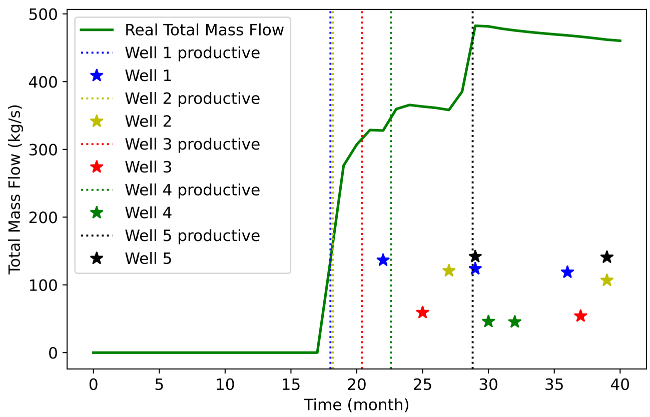
Notes

- The total steam flow is frequently measured at the separator.
- Tracer Flow Test (TFT) can provide an estimate of the well's mass extraction rate at that point in time.
- hese are only taken occasionally for any given well, as the process of taking these measurements is difficult, and may require diverting flow from the separator.

Productive times of individual wells

- Total mass flow into separator is measured frequently.
- Mass flow of individual wells is measured sparsely.
- Productive times of individual wells is known.





Vertical bars indicate the start of well production.

Estimating average well production

$P(t)$: Total output of geothermal power plant at time $t = 1, \ldots, T$

i: Index $i = 1, \ldots, N$ of i^{th} well contributing to the mass flow

$p_i^\star(t)$: Output of well i at time t

$\bar{t}_i$ Start of production of well i

$H(t - \bar{t}_i)$ Heaviside step function indicating productive period of well i

w_i: Average output of well i

$\hat{w}_i$: Estimated average output of well i

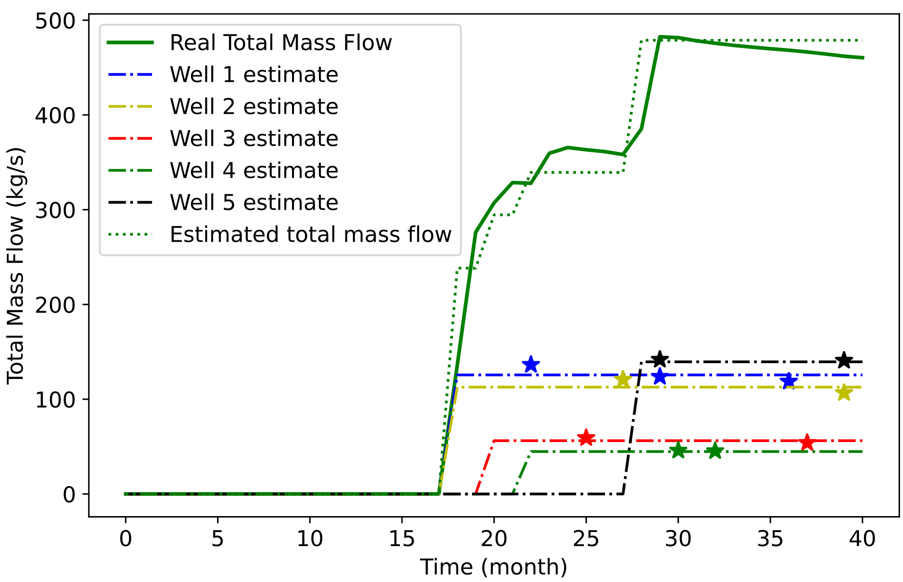
$\delta_{i,t} \in \{0, 1\}$: Binary variable indicating if the output $p_i^\star(t)$ of the i^{th} well was measured ($\delta_{i,t} = 1$) or was not measured ($\delta_{i,t} = 0$) at time t

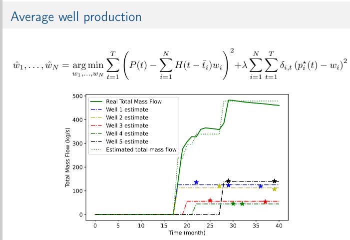
λ: regularisation parameter

$$\hat{w}_1, \ldots, \hat{w}_N = \underset{w_1,\ldots,w_N}{\arg\min} \sum_{t=1}^{T} \left(P(t) - \sum_{i=1}^{N} H(t - \bar{t}_i) w_i \right)^2 + \lambda \sum_{i=1}^{N} \sum_{t=1}^{T} \delta_{i,t} \left(p_i^\star(t) - w_i \right)^2$$

Average well production

$$\hat{w}_1, \ldots, \hat{w}_N = \underset{w_1,\ldots,w_N}{\arg\min} \sum_{t=1}^{T} \left(P(t) - \sum_{i=1}^{N} H(t - \bar{t}_i) w_i \right)^2 + \lambda \sum_{i=1}^{N} \sum_{t=1}^{T} \delta_{i,t} \left(p_i^\star(t) - w_i \right)^2$$





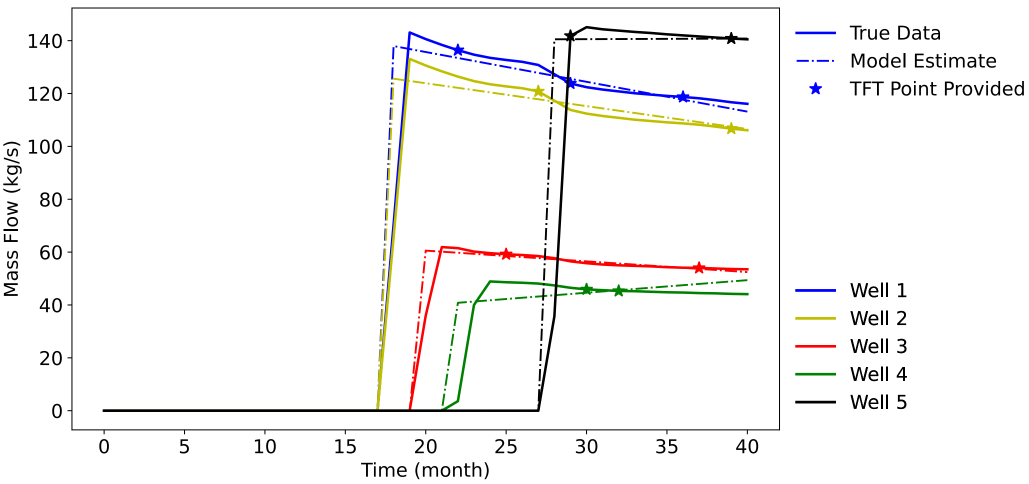
Notes

- Regularisation parameter $\lambda = 93.8$

Piecewise linear well production

$$\mathcal{L} = \sum_{t=1}^{T} \left(P(t) - \sum_{i=1}^{N} H(t - \bar{t}_i) w_i(t) \right)^2 + \lambda \sum_{i=1}^{N} \sum_{t=1}^{T} \delta_{i,t} \left(p_i^{\star}(t) - w_i(t) \right)^2$$

with $w_i(t) = \theta_{i,0} + \theta_{i,1} t$



Forward problem

$p_i(t_j)$: Measured mass flow of i^{th} well at time t_j with $i = 1, \dots, N$ and $j = 1, \dots, T$

$P(t_j)$: Total mass flow into separator plant at time t_j

$\delta_{i,t} \in \{0, 1\}$: indicates if the output $p_i(t_j)$ of the i^{th} well was measured ($\delta_{i,j} = 1$) or not ($\delta_{i,j} = 0$)

$h_{i,j} \in \{0, 1\}$: indicates if i^{th} well was productive at time t_j ($h_{i,j} = 1$) or not productive ($h_{i,j} = 0$)

$\theta_{i,k}$: coefficient of parameterised well output $p_i(t_j) = \sum_{k=0}^{K} \theta_{i,\kappa} t_j^k$

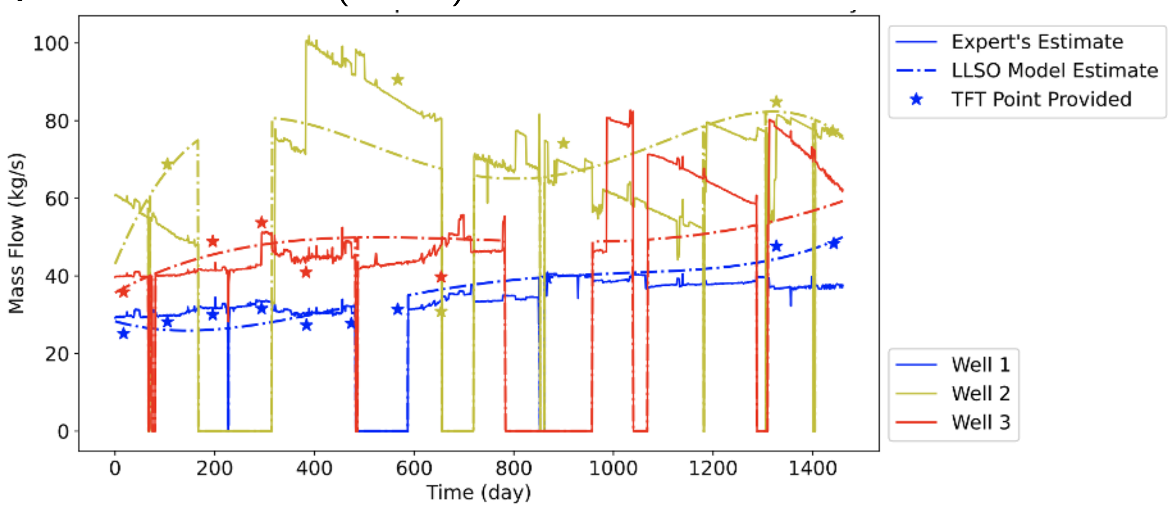
Γ: noise

$$\underbrace{\begin{bmatrix} p_1(t_1)\\ p_1(t_2)\\ \vdots\\ p_1(t_T)\\ p_2(t_1)\\ \vdots\\ \vdots\\ p_N(t_T)\\ P(t_1)\\ P(t_2)\\ \vdots\\ P(t_T)\end{bmatrix}}_{\boldsymbol{y}} = \underbrace{\begin{bmatrix} \delta_{1,1} & 0 & \dots & 0 & 0 & \dots & \dots & 0\\ 0 & \delta_{1,2} & & \vdots & \vdots & & & \vdots\\ \vdots & & \ddots & \vdots & \vdots & & & \vdots\\ 0 & \dots & \dots & \delta_{1,T} & \vdots & & & \vdots\\ 0 & \dots & \dots & \dots & \delta_{2,1} & & & \vdots\\ \vdots & & & & & \ddots & & \vdots\\ \vdots & & & & & & \ddots & \vdots\\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & \delta_{N,T}\\ h_{1,1} & 0 & \dots & 0 & h_{2,1} & 0 & \dots & 0\\ 0 & h_{1,2} & & \vdots & 0 & h_{2,2} & & \vdots\\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots\\ 0 & \dots & \dots & h_{1,T} & 0 & \dots & \dots & h_{N,T} \end{bmatrix}}_{A_1} \underbrace{\underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 & \dots & t_1^K & 0 & \dots & \dots & \dots & 0\\ 1 & t_2 & t_2^2 & \dots & t_2^K & \vdots & & & & \vdots\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & & & & \vdots\\ 1 & t_T & t_T^2 & \dots & t_T^K & 0 & \dots & \dots & \dots & 0\\ 0 & \dots & \dots & \dots & 0 & 1 & t_1 & t_1^2 & \dots & t_1^K\\ \vdots & & & & \vdots & 1 & t_2 & t_2^2 & \dots & t_2^K\\ \vdots & & & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & \dots & \dots & \dots & 0 & 1 & t_T & t_T^2 & \dots & t_T^K \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} \theta_{1,0}\\ \theta_{1,1}\\ \theta_{1,2}\\ \vdots\\ \theta_{1,K}\\ \theta_{2,0}\\ \theta_{2,1}\\ \theta_{2,2}\\ \vdots\\ \theta_{N,K} \end{bmatrix}}_{\boldsymbol{\theta}}}_{(w_1(t_1), w_1(t_2), \dots, w_1(t_T), w_2(t_1), \dots, \dots, w_N(t_T))^{\mathrm{T}}} + \Gamma$$

Use Tikhonov regularisation to solve ill-posed inverse problem!

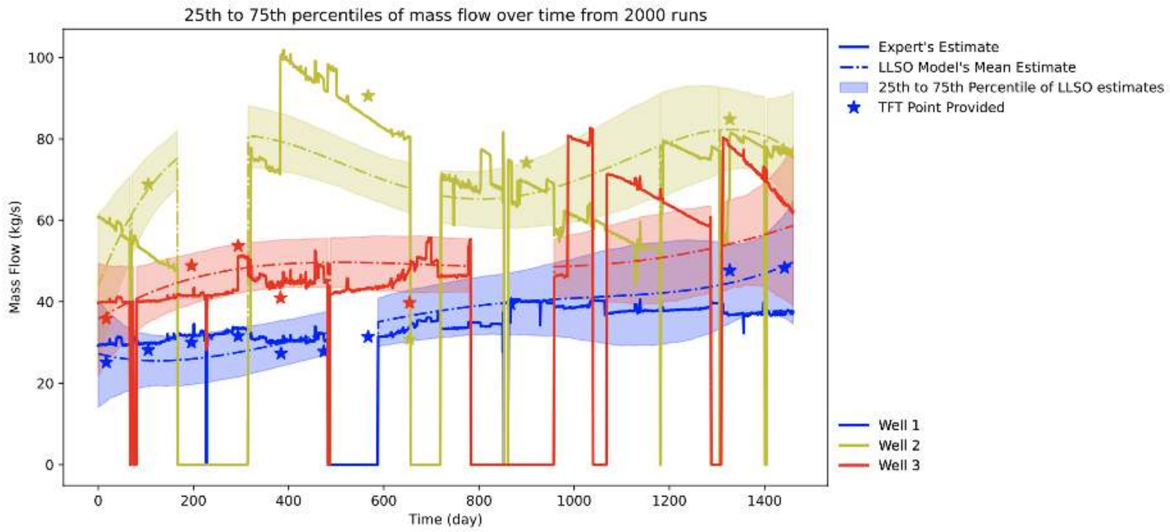
Application to real-world data

- Data provided by Contact Energy.
- Quintic well model ($k = 5$)



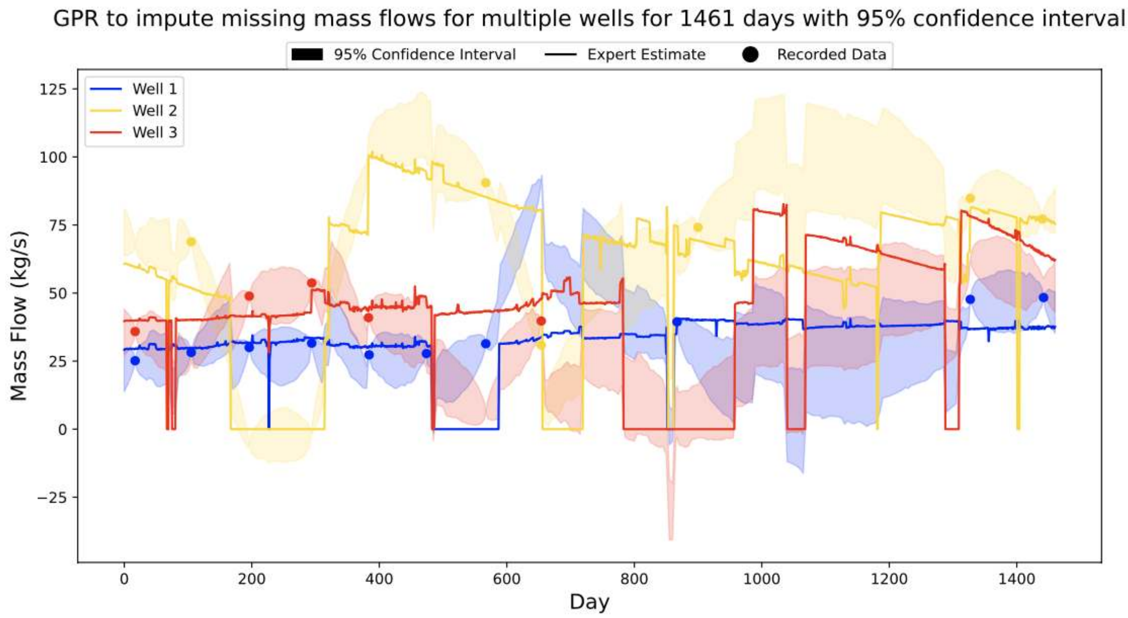
Uncertainty estimation

- Adding Gaussian noise $\mathcal{N}(0, 5)$ to TFT points.
- Estimate well specific mass flow for 2000 realisations.

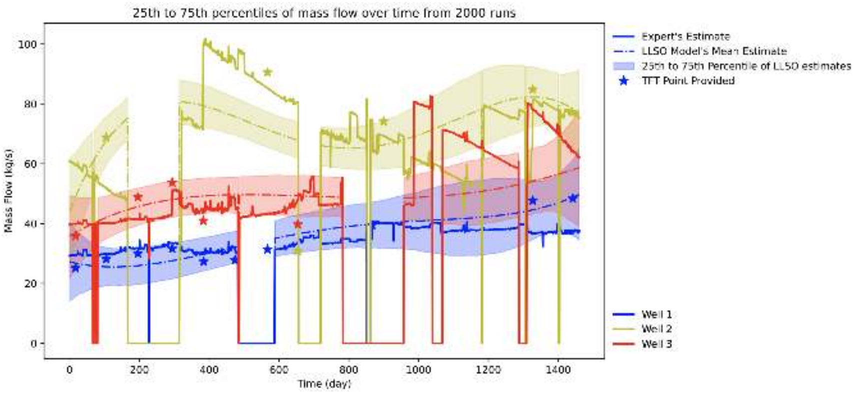


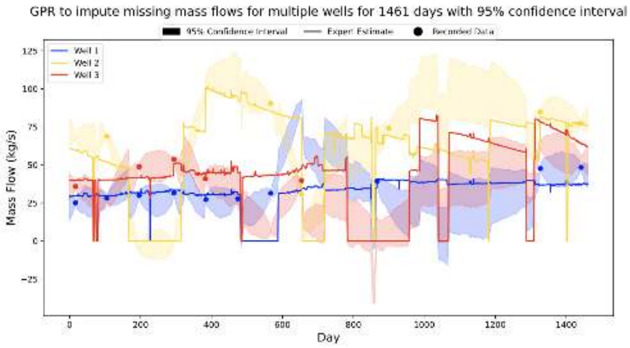
Gaussian Kernel Regression with ccandu

`ccandu` conditional composition and uncertainty quantification



Comparison of uncertainty estimations





Thank you

Olicer Maclaren and Ru Nicholson ill-posed inverse problem

Oliver Maclaren `ccandu`

Paul Abrasaldo and Warren Mannington Well data from Contact Energy